Типови системи

Трифон Трифонов

λ-смятане и теория на доказателствата, 2018/19 г.

7 май 2019 г.

λ-смятане с прости типове

- Типове
  $\rho, \sigma \quad ::= \quad \alpha \mid \rho \Rightarrow \sigma$

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  $(\lambda_x M)N \xrightarrow{\beta} M[x \mapsto N]$
  $(\lambda_x Mx) \xrightarrow{\eta} M$, където $x \notin \mathrm{FV}(M)$

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λ-смятане с прости типове

$c_n : (\alpha \Rightarrow \alpha) \Rightarrow \alpha \Rightarrow \alpha$

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- Силно нормализируема
- Изразителна сила: разширени полиноми
  $$p(x) \quad ::= \quad 0 \mid 1 \mid x \mid p_1(x) + p_2(x) \mid p_1(x).p_2(x) \mid p_1(p_2(x)) \mid \min(x,1) \mid 1 - \min(x,1)$$

λ-смятане с произведение

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- Редукции
  $$(\lambda_x M)N \xrightarrow{\beta} M[x \mapsto N] \qquad \langle M, N\rangle\llcorner \xrightarrow{\beta} M \qquad \langle M, N\rangle\lrcorner \xrightarrow{\beta} N$$
  $(\lambda_x Mx) \xrightarrow{\eta} M$, където $x \notin \mathrm{FV}(M)$ $\qquad \langle M\llcorner, M\lrcorner\rangle \xrightarrow{\eta} M$

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  $(\lambda_x M)N \xrightarrow{\beta} M[x \mapsto N] \qquad \langle M, N \rangle \llcorner \xrightarrow{\beta} M \qquad \langle M, N \rangle \lrcorner \xrightarrow{\beta} N$
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  $(\lambda_x M)N \xrightarrow{\beta} M[x \mapsto N] \qquad \langle M, N \rangle \llcorner \xrightarrow{\beta} M \qquad \langle M, N \rangle \lrcorner \xrightarrow{\beta} N$
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  $(\lambda_x M)N \xrightarrow{\beta} M[x \mapsto N] \qquad \langle M, N\rangle\llcorner \xrightarrow{\beta} M \qquad \langle M, N\rangle\lrcorner \xrightarrow{\beta} N$
  $(\lambda_x Mx) \xrightarrow{\eta} M$, където $x \notin \mathrm{FV}(M) \qquad \langle M\llcorner, M\lrcorner\rangle \xrightarrow{\eta} M$
- Силно нормализируема
- Изразителна сила: не се променя
- Произведението е “синтактична захар”

Система на Hindley-Milner

- Монотипове

$$\rho, \sigma \quad ::= \quad \alpha \mid \rho \Rightarrow \sigma$$

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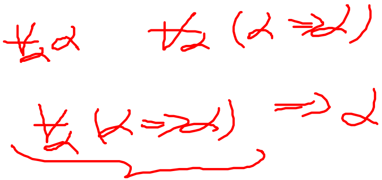
  $\rho, \sigma \quad ::= \quad \alpha \mid \rho \Rightarrow \sigma$

- Политипове

  $\tau \quad ::= \quad \sigma \mid \forall_\alpha \tau$



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- Безтипови термове

  $M, N \quad ::= \quad x \mid MN \mid \lambda_x M \mid \textbf{let } x = M \textbf{ in } N$

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  $M, N \quad ::= \quad x \mid MN \mid \lambda_x M \mid \textbf{let } x = M \textbf{ in } N$
- Наредба на типовете

$$\frac{\sigma_2 \equiv \sigma_1[\vec{\alpha} \mapsto \vec{\rho}]}{\forall_{\vec{\alpha}} \sigma_1 \sqsubseteq \forall_{\vec{\beta}} \sigma_2}\ \vec{\beta} \notin FV(\forall_{\vec{\alpha}} \sigma_1)$$

$\sigma_2 = \sigma_1[\alpha \mapsto \beta \Rightarrow \beta]$

$\sigma_1 = \alpha \Rightarrow \alpha$

$\sigma_2 = (\beta \Rightarrow \beta) \Rightarrow \beta \Rightarrow \beta$

$\forall_\alpha (\alpha \Rightarrow \alpha) \sqsubseteq \forall_\beta \sigma_2$

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- Редукции
  $(\lambda_x M)N \xrightarrow{\beta} M[x \mapsto N] \qquad \textbf{let } x = N \textbf{ in } M \xrightarrow{\beta} M[x \mapsto N]$
  $(\lambda_x Mx) \xrightarrow{\eta} N$, където $x \notin \mathrm{FV}(M)$

Система на Hindley-Milner: типова изводимост

$$\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \qquad \frac{\Gamma, x : \rho \vdash M : \sigma}{\Gamma \vdash \lambda_x M : \rho \Rightarrow \sigma}$$

$$\frac{x : \alpha \vdash x : \alpha}{\vdash \lambda_x x : \alpha \Rightarrow \alpha}$$

$$\vdash \lambda_x x : \forall_\alpha (\alpha \Rightarrow \alpha)$$

$$\frac{\Gamma \vdash M : \rho \Rightarrow \sigma \qquad \Gamma \vdash N : \rho}{\Gamma \vdash MN : \sigma}$$

$$\frac{\Gamma \vdash M : \tau \qquad \Gamma, x : \tau \vdash N : \sigma}{\Gamma \vdash \mathbf{let}\ x = M\ \mathbf{in}\ N : \sigma}$$

$$\frac{\Gamma \vdash M : \tau_1 \qquad \tau_1 \sqsubseteq \tau_2}{\Gamma \vdash M : \tau_2} \qquad \frac{\Gamma \vdash M : \tau}{\Gamma \vdash M : \forall_\alpha \tau}\ \alpha \notin FV(\Gamma)$$

$$\forall_\alpha (\alpha \Rightarrow \alpha) \sqsubseteq \alpha_1 \Rightarrow \alpha_1$$

$$\sqsubseteq \alpha_2 \Rightarrow \alpha_2$$

- Типовата изводимост е разрешима (Алгоритъм W)

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$$\frac{\Gamma \vdash M : \rho \Rightarrow \sigma \qquad \Gamma \vdash N : \rho}{\Gamma \vdash MN : \sigma}$$

$$\frac{\Gamma \vdash M : \tau \qquad \Gamma, x : \tau \vdash N : \sigma}{\Gamma \vdash \mathbf{let}\ x = M\ \mathbf{in}\ N : \sigma}$$

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Programming Computable Functionals (PCF)

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  $\rho, \sigma \quad ::= \quad \texttt{nat} \mid \texttt{bool} \mid \rho \Rightarrow \sigma \mid \rho \otimes \sigma$

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& \langle M^{\rho}, N^{\sigma} \rangle^{\rho \otimes \sigma} \mid \mathsf{Split}_{\rho,\sigma,\tau} : \rho \otimes \sigma \Rightarrow (\rho \Rightarrow \sigma \Rightarrow \tau) \Rightarrow \tau \mid \\
& \mathsf{tt}^{\texttt{bool}} \mid \mathsf{ff}^{\texttt{bool}} \mid 0^{\texttt{nat}} \mid \mathsf{S}^{\texttt{nat} \Rightarrow \texttt{nat}} \mid \mathsf{P}^{\texttt{nat} \Rightarrow \texttt{nat}} \mid \mathsf{Z}^{\texttt{nat} \Rightarrow \texttt{bool}} \mid \\
& \mathsf{Cases}_{\tau} : \texttt{bool} \Rightarrow \tau \Rightarrow \tau \Rightarrow \tau \mid \mathsf{Y}_{\tau} : (\tau \Rightarrow \tau) \Rightarrow \tau
\end{aligned}
$$

Programming Computable Functionals (PCF): редукции

- Редукции

$$\begin{array}{llllll}(\lambda_x s)t & \stackrel{\beta}{\to} & s[x \mapsto t], & \mathrm{P}\,0 & \stackrel{\beta}{\to} & 0,\\ \mathrm{Split}\,\langle s, t\rangle\, f & \stackrel{\beta}{\to} & f\ s\ t, & \mathrm{P}\,(\mathrm{S}n) & \stackrel{\beta}{\to} & n\end{array}$$

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\mathsf{Cases}\,\mathsf{tt}\ s\ t & \xrightarrow{\beta} & s, & \mathsf{Z}\,0 & \xrightarrow{\beta} & \mathsf{tt}, \\
\mathsf{Cases}\,\mathsf{ff}\ s\ t & \xrightarrow{\beta} & t, & \mathsf{Z}\,(\mathsf{S}n) & \xrightarrow{\beta} & \mathsf{ff},
\end{array}$$

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\mathrm{Cases}\,\mathrm{ff}\ s\ t &\stackrel{\beta}{\to} t, & \mathrm{Z}\,(\mathrm{S}n) &\stackrel{\beta}{\to} \mathrm{ff},\\
\mathrm{Y}\ t &\stackrel{\beta}{\to} t(\mathrm{Y}\ t)
\end{aligned}
$$

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\mathsf{Cases}\,\mathsf{tt}\; s\; t & \stackrel{\beta}{\to} & s, & \mathsf{Z}\,0 & \stackrel{\beta}{\to} & \mathsf{tt}, \\
\mathsf{Cases}\,\mathsf{ff}\; s\; t & \stackrel{\beta}{\to} & t, & \mathsf{Z}\,(\mathsf{S}n) & \stackrel{\beta}{\to} & \mathsf{ff}, \\
\mathsf{Y}\; t & \stackrel{\beta}{\to} & t(\mathsf{Y}\; t) & & &
\end{array}
$$

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\mathrm{Cases}\,\mathrm{tt}\, s\, t & \xrightarrow{\beta} & s, & \mathrm{Z}\,0 & \xrightarrow{\beta} & \mathrm{tt}, \\
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\mathrm{Y}\, t & \xrightarrow{\beta} & t(\mathrm{Y}\, t) & & &
\end{array}
$$

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- Изразителна сила: всички изчислими функции

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- Типове

$\rho, \sigma \quad ::= \quad \texttt{nat} \mid \texttt{bool} \mid \rho \Rightarrow \sigma \mid \rho \otimes \sigma$

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Система T на Gödel: редукции

- Редукции

$$
\begin{array}{lcl}
(\lambda_x s)t & \xrightarrow{\beta} & s[x \mapsto t], \\
\mathsf{Split}\, \langle s, t\rangle\, f & \xrightarrow{\beta} & f\, s\, t,
\end{array}
\qquad
\begin{array}{lcl}
\mathsf{Cases}\,\mathsf{tt}\, s\, t & \xrightarrow{\beta} & s, \\
\mathsf{Cases}\,\mathsf{ff}\, s\, t & \xrightarrow{\beta} & t,
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$$

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$$\begin{array}{lcl} (\lambda_x s)t & \xrightarrow{\beta} & s[x \mapsto t], \\ \mathsf{Split}\,\langle s,t\rangle\, f & \xrightarrow{\beta} & f\,s\,t, \\ \mathcal{R}\,0\,s\,t & \xrightarrow{\beta} & s, \\ \mathcal{R}\,(\mathsf{S}n)\,s\,t & \xrightarrow{\beta} & t\,n\,(\mathcal{R}\,n\,s\,t). \end{array} \qquad \begin{array}{lcl} \mathsf{Cases}\,\mathsf{tt}\,s\,t & \xrightarrow{\beta} & s, \\ \mathsf{Cases}\,\mathsf{ff}\,s\,t & \xrightarrow{\beta} & t, \end{array}$$

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(\lambda_x s)t &\xrightarrow{\beta} s[x \mapsto t], & \mathrm{Cases}\,\mathrm{tt}\,s\,t &\xrightarrow{\beta} s,\\
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\mathcal{R}\,0\,s\,t &\xrightarrow{\beta} s, & &\\
\mathcal{R}\,(\mathrm{S}n)\,s\,t &\xrightarrow{\beta} t\,n\,(\mathcal{R}\,n\,s\,t). & &
\end{aligned}$$

- Силно нормализируема

Система T на Gödel: редукции

- Редукции

$$\begin{array}{lcl} (\lambda_x s)t & \stackrel{\beta}{\to} & s[x \mapsto t], \\ \mathrm{Split}\,\langle s, t\rangle\, f & \stackrel{\beta}{\to} & f\, s\, t, \\ \mathcal{R}\, 0\, s\, t & \stackrel{\beta}{\to} & s, \\ \mathcal{R}\, (\mathrm{S}n)\, s\, t & \stackrel{\beta}{\to} & t\, n\, (\mathcal{R}\, n\, s\, t). \end{array} \qquad \begin{array}{lcl} \mathrm{Cases}\,\mathrm{tt}\, s\, t & \stackrel{\beta}{\to} & s, \\ \mathrm{Cases}\,\mathrm{ff}\, s\, t & \stackrel{\beta}{\to} & t, \end{array}$$

- Силно нормализируема
- Изразителна сила: доказуемо тотални функции