

Finite Element Method—1

Additional Problems. Part 3.

Elementwise computations

Problem 1. Working elementwise, derive algorithms for assembling the linear algebraic systems, derived in “Additional Problems. Part 1” and “Additional Problems. Part 2.”.

FEM for non-stationary problems

Problem 2. Formulate FEM for solving approximately the following non-stationary differential problem, thus, obtaining the corresponding semi-discrete problem (an ODE system). By using a time discretization method, construct a fully-discrete numerical scheme.

$$\begin{aligned}\frac{\partial u}{\partial t} - \nabla \cdot (\sin(x+y)\nabla u) + 10u &= x, & (x, y) \in \Omega, & t > 0, \\ u(x, y, 0) &= u_0(x, y), & (x, y) \in \Omega, \\ u(x, y, t) &= 0, & (x, y) \in \Gamma_1, \\ -\sin(x+y)\nabla u \cdot \mathbf{n} &= 10, & (x, y) \in \Gamma_2.\end{aligned}$$