ДАА

упр.№5, 30 април 2020г.

Hello, world!

Решаване на рек. уравн.

- разбиване
- дърво на рекурсията
- индукция
- характеристичното уравн.
- Master Theorem

$T(n) = \ldots$

$T(\ldots)$

$$\begin{cases} n-c \rightarrow \frac{n}{c} = \Theta(n) \\ \frac{n}{c} \rightarrow \log_c n = \Theta(\log n) \\ \sqrt{n} \rightarrow \Theta(\log \log n) \end{cases}$$

Rezolvare

$\cancel{T(c) = \Theta(1)}$

$T(1) = \Theta(1)$

$$T(n) = \underbrace{\underbrace{T(n-1)}_{} + \lg n}_{} =$$

$$= T(n-2) + \lg(n-1) + \lg n =$$

$$= T(n-3) + \lg(n-2) + \lg(n-1) + \lg n =$$

$$= T(n-4) + \lg(n-3) + \lg(n-2) + \lg(n-1) + \lg n =$$

$$= \ldots = T(1) + \lg 2 + \lg 3 + \ldots + \lg n =$$

$$= \Theta(1) + \lg(2 \cdot 3 \cdot 4 \ldots (n-1) \cdot n) = \Theta(1) + \lg n! \simeq \Theta(1) + \Theta(n \lg n)$$

$$= \Theta(n \lg n)$$

$$T(n) = 2\underbrace{T\left(\frac{n}{2}\right)} + 1 = 2\left(2T\left(\frac{n}{4}\right) + 1\right) + 1$$

$$\sum_{i=0}^{k} 2^k = 2^{k+1} - 1$$

Нека $n = 2^k$ за $k \in \mathbb{N}$

$$= 4\underbrace{T\left(\frac{n}{4}\right)}_{2T\left(\frac{n}{8}\right)+1} + 2 + 1$$

$$= 8T\left(\frac{n}{8}\right) + 4 + 2 + 1$$

$$= 16T\left(\frac{n}{16}\right) + 8 + 4 + 2 + 1 =$$

$$= \dots = nT\left(\frac{n}{n}\right) + \frac{n}{2} + \frac{n}{4} + \dots + 8 + 4 + 2 + 1$$

$$= \underbrace{n.\Theta(1)}_{n} + \underbrace{\frac{n}{2} + \frac{n}{4} + \dots + 4 + 2 + 1}_{n-1} = n.\Theta(1) + n - 1 = \Theta(n)$$

$$T(n) = \Omega(n) \Leftarrow n \le n\left(1 + \frac{1}{2} + \frac{1}{4} + \dots + \frac{1}{n}\right) = n\left(1 + \frac{1}{2^1} + \frac{1}{2^2} \dots + \frac{1}{2^{\lg n}}\right) \le n\left(1 + \frac{1}{2} + \dots\right) \le n.c$$

Сума на геом. пр. с $q < 1$ $\Rightarrow T(n) = O(n)$

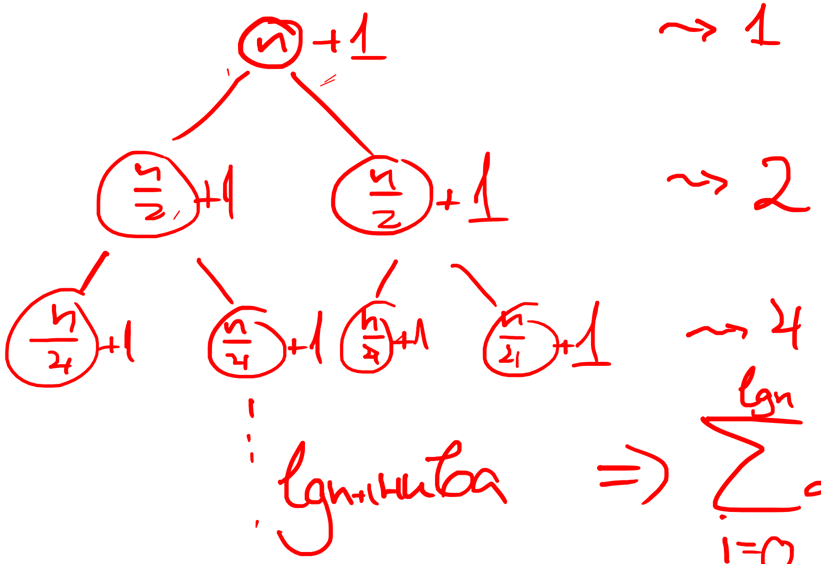
$$\Rightarrow T(n) = \Theta(n)$$

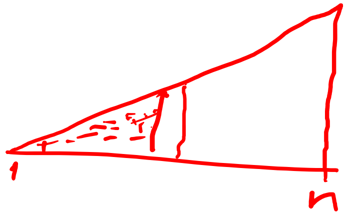
Домашно: с развиване $T(n) = n.T(n-1) + 1$

——— " ——— $T(n) = T(n-2) + 2\lg n$

с индукция $T(n) = nT(n-1) + 1$

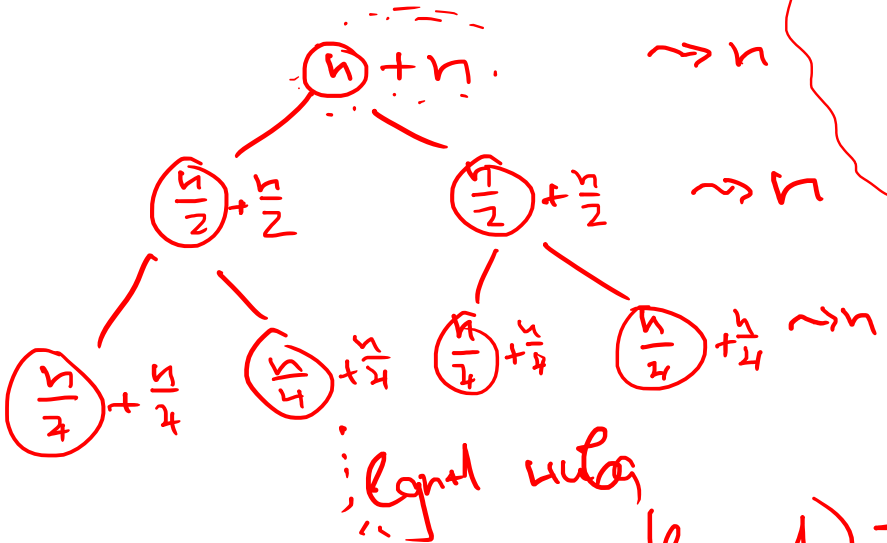
$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$





$$\Rightarrow \sum_{i=0}^{\lg n} 2^i = 2^{\lg n+1} - 1 = 2n-1 = \Theta(n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + n$$



$$\Rightarrow T(n) = n.(\lg n+1) = \Theta(n \lg n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1 \overset{?}{=} \Theta(n)$$

Ще док. по индукция, че $T(n) = O(n)$.

$\Leftrightarrow$ ще док., че $\exists c > 0 \; \exists n_0 > 0 : \forall n \geq n_0 \quad T(n) \leq c.n$

Нека това е вярно за $\forall n' < n$. Ще док., че $T(n) \leq c.n$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1 \overset{\text{ИП}}{\leq} 2\left(c \cdot \frac{n}{2}\right) + 1 = c.n + 1 \overset{?}{\leq} c.n$$

$\Rightarrow$ НЕ ?!?!

Ще подсилим тв.: ще док., че $\exists c > 0$ и $\exists b > 0$ т.че $T(n) \leq c.n - b$

(оттам ще следва, че $T(n) \leq c.n$)

$$T(n) = 2T\left(\frac{n}{2}\right) + 1 \overset{\text{ИП}}{\leq} 2\left(c\frac{n}{2} - b\right) + 1 = \cancel{c.n} - 2b + 1 \overset{?}{\leq} \cancel{c.n}$$

$\Leftrightarrow \overset{?}{\exists} b > 0 : -2b + 1 \leq 0 \Leftrightarrow 2b \geq 1 \Rightarrow$ да, напр. $b = \frac{1}{2}$

и c-произв.

$$\Rightarrow T(n) = O(n)$$

Ще док. че $\exists d>0\ \exists n_0 \geq 0 : \forall n \geq n_0\ T(n) \geq d.n$

$$T(n) = 2T\left(\frac{n}{2}\right)+1 \overset{ИП}{\geq} 2.\left(d\cdot\frac{n}{2}\right)+1 = dn+1 \overset{?}{\geq} d.n$$

$\rightarrow$ отново $\forall d$ върши работа

$\Rightarrow T(n) = \Omega(n)$

$\Rightarrow$ оконч. $T(n) = \Theta(n)$

$$T(n) = 2T(n-1) + n \overset{?}{\Rightarrow} \Theta(2^n)$$

I) ще док. че $\exists c>0 \; \exists n_0 \; \forall n \ge n_0 \quad T(n) \le c.2^n$

$$T(n) = 2T(n-1) + n \overset{ИП}{\le} 2(c.2^{n-1}) + n =$$

$$= c.2^n + n \overset{?}{\le} c.2^n \Rightarrow \text{не} \dots$$

$$\exists b>0 \quad T(n) \le c.2^n - b$$

$$T(n) = \dots \overset{ИП}{\le} 2(c.2^{n-1} - b) + n = c.2^n - b + n \le c.2^n - b$$

$$\overset{?}{\iff} \exists b>0 : n \le 0$$

$\to$ не – за никое b

$$\dots \quad \exists b>0 : T(n) \le c.2^n - b.n$$

$$T(n) = \dots \overset{ИП}{\le} 2(c.2^{n-1} - b(n-1)) + n = c2^n - 2bn + 2b + n \overset{?}{\le} c2^n - bn$$

$$\overset{?}{\iff} \exists b>0 : 2b \le (b-1)n \iff n \ge \frac{2b}{b-1}$$

$\to$ да, напр. за $b=2$ при дост. големи n

$$n \ge \frac{2b}{b-1}$$

$\Rightarrow$ за $\forall c>0$

$$T(n) \le c2^n - bn \le c.2^n$$

$$\Rightarrow T(n) = O(2^n)$$