ДАА

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Hello, world!

...от миналия път

- $T(n) = n * T(n-1) + 1 = n((n-1).T(n-2)+1)+1 =$

$= n.(n-1)T(n-2)+n+1 =$

$= n(n-1)(n-2)T(n-3)+n(n-1)+n+1 =$

$= n(n-1)(n-2)(n-3)T(n-4)+n(n-1)(n-2)+n(n-1)+n+1$

$= \dots = n.(n-1)\dots 4.3.2.T(1)+n.(n-1)\dots 4.3+n(n-1)\dots 4+\dots+n(n-1)+n+1$

$= n!T(1)+\frac{n!}{2!}+\frac{n!}{3!}+\frac{n!}{4!}+\dots+\frac{n!}{(n-2)!}+\frac{n!}{(n-1)!}+\frac{n!}{n!} =$

$= n!\Theta(1)+n!\underbrace{\sum_{i=2}^{n}\frac{1}{i!}}_{\text{огр. от конст.}} < n!\Theta(1)+n!.const = O(n!)$

$\geq n!\Theta(1) \Rightarrow T(n)=\Omega(n!) \Rightarrow T(n)=\Theta(n!)$

- $T(n) = T(n-2) + 2 * \lg n$

$\lim_{n\to\infty}\frac{\frac{1}{(n+1)!}}{\frac{1}{n!}} = \lim_{n\to\infty}\frac{n!}{(n+1)!} = \lim\frac{1}{n+1} = 0 < 1 \Rightarrow \sum_{i=0}^{\infty}\frac{1}{i!} < const$

$$T(n) = T(n-2) + 2\lg n =$$

$$= T(n-4) + 2\lg(n-2) + 2\lg n =$$

$$= T(n-6) + 2\lg(n-4) + 2\lg(n-2) + 2\lg n$$

Inc. $= \dots = T(1) + 2\lg 3 + \dots 2\lg(n-2) + 2\lg n$

n - нечетно

$$= \Theta(1) + \lg 3 + \lg 3 + \lg 5 + \lg 5 + \dots + \lg(n-2) + \lg(n-2) + \lg n + \lg n = S$$

$$S > \lg 2 + \lg 3 + \lg 4 + \lg 5 + \dots + \lg(n-3) + \lg(n-2) + \lg(n-1) + \lg n$$

$$\Leftrightarrow S > \lg(n!) \asymp n\lg n \Rightarrow S = \Omega(n\lg n)$$

$$S < \lg 3 + \lg 4 + \lg 5 + 6 + \dots + \lg n + \lg(n+1)$$

$$S < \lg(n+1)! - \lg 2 = \Theta((n+1)\lg(n+1)) = \Theta(n\lg n)$$

$$\Rightarrow S = O(n\lg n) \Rightarrow S = \Theta(n\lg n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$$

разбиване?

$$T(n) = \underline{A_1} . T(n - \underline{c_1}) + A_2 . T(n-c_2) + \dots + \underline{b_1^n} . \underline{P_1(n)} + \underline{b_2^n} P_2(n) + \dots$$

x^n const const const полином

$b_1, b_2, \dots \Rightarrow$ уникални

$$x^n = A_1 . x^{n-c_1} + A_2 . x^{n-c_2} + \dots$$

$$+ 2^n + n . 2^n = \dots + 2^n(n+1)$$

$\lim \frac{3^n}{2^n . n^2} = \lim \frac{\left(\frac{3}{2}\right)^n}{n^2} = \infty$

заради общата

$$T(n) = T(n-1) + 2T(n-2)$$

$$x^n = x^{n-1} + 2x^{n-2}$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$D = 1 + 8 = 9$$

$$x_1 = \frac{1-3}{2} = -1$$

$$x_2 = \frac{1+3}{2} = 2$$

$\{-1; 2\}_м$

всяко b_i включване толкова пъти, колкото е $deg(P_i) + 1$

$$+ 2^n(n+1) + 3^n(n^5 + n^4 + 1) + 1^n(n+1) + 4^n . n^0$$

$\{2; 2\}$ $\{3, 3, 3, 3, 3, 3\}$ $\{1; 1\}$ $\{4\}$

Нека откт. имаме $\{1; 2; 2; 3\}$

$$T(n) = A 1^n + B . 2^n + C . 2^n . n + D . 2^n . n^2 + E . 3^n = \Theta(3^n)$$

1. $T(n) = T(n-1) + 1$
2. $T(n) = T(n-2) + 2n$ $.1^n$
3. $T(n) = 2T(n-1) + 2^n$
4. $T(n) = T(n-1) + T(n-2)$
5. $T(n) = 2T(n-1) - T(n-2)$
6. $T(n) = 4T(n-2) + 3^n$ $.n^0$ $\rightarrow x^n = 4x^{n-2} \Leftrightarrow x^2 - 4 = 0$ $\{-2;2\}$ $\Rightarrow \{-2;2;3\}$ $\rightarrow \{3\}$

   $T(n) = \underbrace{A.(-2)^n}_{\Theta(2^n)} + B.2^n + C.3^n = \Theta(3^n)$ $\quad \Theta(2^n) \quad \Theta(3^n)$
7. $T(n) = 3T(n-1) - 2T(n-2) + n2^n$ $\Rightarrow x^2 - 3x + 2 = 0$ $\Rightarrow \{1;2\}$ $\Rightarrow \{1;2;2;2\}$

   $n2^n \downarrow \{2;2\}$

   $\Rightarrow T(n) = A.1^n + B.2^n + C.n2^n + D.n^2.2^n$
   $= \Theta(n^2.2^n)$
8. $T(n) = \sum_{i=1}^{n-1} T(i) + 2^n$
9. $T(n) = \sum_{i=1}^{n-2} T(i) + 3^{\frac{n}{2}}$
10. $T(n) = 2T\left(\frac{n}{2}\right) + 1$

$$T(n) = \sum_{i=1}^{n-1} T(i) + 2^n$$
$$-\quad T(n-1) = \sum_{i=1}^{n-2} T(i) + 2^{n-1}$$
$$T(n) - T(n-1) = T(n-1) + 2^n - 2^{n-1} \Leftrightarrow T(n) = 2T(n-1) + \frac{1}{2}.2^n$$

$x = 2 \qquad \{2\}$

$$\Rightarrow T(n) = A.2^n + Bn2^n = \Theta(n.2^n)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$n = 2^m$ $\qquad T(n) = T(2^m) = S(m)$

$\lg n = m$

$$\frac{n}{2} = \frac{2^m}{2} = 2^{m-1}$$

$$T\left(\frac{n}{2}\right) = T(2^{m-1}) = S(m-1)$$

$$\Rightarrow S(m) = 2S(m-1) + \underset{\{1\}}{1 \cdot 1^n}$$

$x = 2 \Rightarrow \{2\}$

$\rightarrow \{1;2\} \Rightarrow S(m) = A \cdot 1^m + B \cdot 2^m$

$$T(n) = S(m) = \Theta(2^m) = \Theta(n)$$

$$T(n) = a \cdot T\left(\frac{n}{b}\right) + f(n)$$

$k = \log_b a$. (сравн. $f(n)$ с n^k)

<u>I сл.</u> За $\varepsilon > 0$ $n^{k-\varepsilon} \succsim f(n) \Rightarrow T(n) \asymp n^k$

<u>II сл.</u> $f(n) \asymp n^k \cdot \lg^t n$ за $t \geq 0 \Rightarrow T(n) \asymp n^k \cdot \lg^{t+1} n$

<u>III сл.</u> За $\varepsilon > 0$ $n^{k+\varepsilon} \precsim f(n) \Rightarrow T(n) \asymp f(n)$

и за $c \in (0;1)$ е вярно, че за дост. големи n

$$a \cdot f\left(\frac{n}{b}\right) < c \cdot f(n)$$

1. $T(n) = T\left(\frac{9n}{10}\right) + n$

2. $T(n) = 16T\left(\frac{n}{4}\right) + n^2$

3. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

4. $T(n) = 4T\left(\frac{n}{2}\right) + n$

5. $T(n) = 3T\left(\frac{n}{2}\right) + n \Rightarrow$ Karatsuba

6. $T(n) = 3T\left(\frac{n}{4}\right) + n\lg n$

7. $T(n) = 8T\left(\frac{n}{2}\right) + n^3$

8. $T(n) = 2T\left(\frac{n}{2}\right) + n \Rightarrow$ MergeSort

9. $T(n) = 2T\left(\frac{n}{2}\right) + \lg n$

10. $T(n) = 2T\left(\frac{n}{2}\right) + n\lg n$

11. $T(n) = 5T\left(\frac{n}{2}\right) + n^2\lg n$

12. $T(n) = 2T\left(\frac{n}{2}\right) + 1$

13. $T(n) = 2T(\sqrt{n}) + \lg n$

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4. $k = \log_2 4 = 2$

Сравн. $n^k = n^2$ с $f(n) = n$

$\exists^{?}\ \varepsilon > 0:\ n^{k-\varepsilon} \succeq f(n)$

– за $\forall \varepsilon \in (0;1)$ е вярно, че $2 - \varepsilon \geq 1$

$\Longleftrightarrow n^{k-\varepsilon} \succeq \cancel{n} = f(n)$

$\Rightarrow$ по 1сл. на МТ

$T(n) \asymp n^2$

3. $T(n) = 2T\left(\frac{n}{4}\right) + \sqrt{n}$

$k = \log_4 2 = \frac{1}{2} \Rightarrow$ сравн. $n^k = n^{\frac{1}{2}}$ с $f(n) = n^{\frac{1}{2}}$

$\Rightarrow f(n) = n^k \Rightarrow f(n) \asymp n^k \cdot \lg^t n$ за $t = 0$

$\Rightarrow$ по II сл. на МТ $T(n) \asymp \sqrt{n} \cdot \lg n$

$\log_a b = \frac{1}{\log_b a}$

Сравн. n^k с $f(n) = 2^n$

за $\forall \varepsilon > 0$ $n^{k+\varepsilon} \prec 2^n$

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6. $T(n) = 3T\left(\frac{n}{4}\right) + n\lg n$

$k = \log_4 3 \in (0;1) \Rightarrow$ сравн. n^k с $f(n) = n . \lg n$

$\Rightarrow$ за $\forall \varepsilon \in (0, 1 - \log_4 3)$ е вярно, че $n^{k+\varepsilon} \prec n \prec n\lg n$

Търсим $c \in (0;1)$: $\exists n_0 > 0 : \forall n \geq n_0$ $3 \cdot \frac{n}{4} \cdot \lg\frac{n}{4} < c . n \lg n$

$\Leftrightarrow \exists ? c : \frac{3}{4} . n . (\lg n - 2) < c . n . \lg n \;\Big|\; : n$

$\Leftrightarrow \exists ? c : \frac{3}{4} \cdot \lg n - \frac{3}{2} < c . \lg n$

$\rightarrow$ да, за $c = \frac{3}{4}$ за $\forall n > 0$ това е вярно

по III сл. на МТ $\Rightarrow T(n) \asymp n \lg n$