The actual experimental design

- It is usually presented as a Table with respective columns and rows
- The rows represent the number of necessary experiments to complete the design
- The columns indicate the values of the inputs responsible for performing the real experiments; several specific columns are also added to the design – column for the intercept, columns for the repeated experiments (output function values); sometime additional columns are included to reflect the calculation algorithm for each term of the final model

The simple case FFD 2^2

Exp. Number	X0	X1	X2	X1X2	Y	SD
1	+ 1	+ 1	+ 1	+ 1	Y1	SD1
2	+ 1	-1	+ 1	-1	Y2	SD2
3	+ 1	+ 1	-1	-1	Y3	SD3
4	+ 1	-1	-1	+ 1	Y4	SD4

The statistical model

$$Y = a_0 + a_1X_1 + a_2X_2 + a_{12}X_1X_2 + E$$

The model reveals the relationship between the output and each one of the input factors as well as of their mixed effect

The simple calculation algorithm

- a_0 is the intercept: sum of all Y_i with *+ sign* divided by n (number of experiments); it indicates an average of the output when the inputs are on the middle of the experimental interval (formally Xi = 0 but it does not mean "lack of input factors");
- a_i are single regression coefficients whose values and signs show the weight (impact) of each input factor on the output function; their calculation could be presented as sum of all Y_i values with sign corresponding to the indication in the respective column of the design divided by the number of experiments;

Other calculations

- a_{ij} are mixed regression coefficient indicating the simultaneous impact of two (or more in more complex designs) input factors on the output function – again, by their value and sign ; the calculation follows the same pattern as in calculation of single regression coefficients but the sign of the output is determined by multiplication of the signs for two columns from the design;
- In case of more sophisticated designs (with more input factors) except the mixed effect for two factors, impacts of the effect of three factors simultaneously, four factors etc, should be calculated and interpreted

General rules for FFD 2^n

- The total number of experiments needed for the statistical model is

$$N = 2^n$$

where n is the number of input factors;

- Each real experiment should be replicated in order to estimate the experimental error;
- The experiment realization should follow the principle of randomization (not obligatory from 1 to N but in random order); it avoids effects of "block" impact (too many experiments with + or – signs one after another);

Design 2^3

No	Factor 1	Factor 2	Factor 3	Y
1	+1	+1	+1	y_1
2	-1	+1	+1	y_2
3	+1	-1	+1	y_3
4	-1	-1	+1	y_4
5	+1	+1	-1	y_5
6	-1	+1	-1	y_6
7	+1	-1	-1	y_7
8	-1	-1	-1	y_8

n

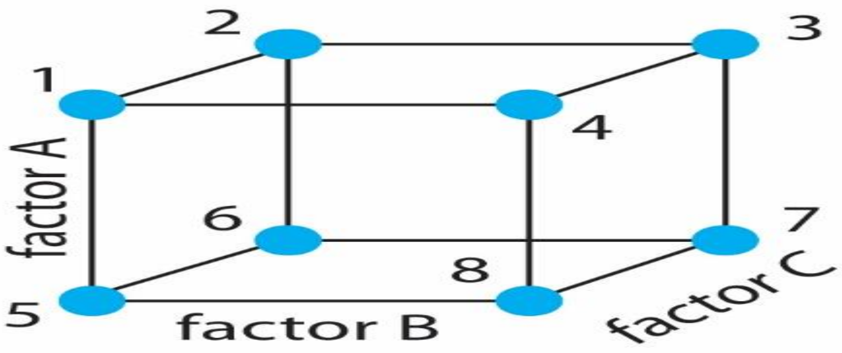
$$Y = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_{12} X_1 X_2 + a_{13} X_1 X_3 + a_{23} X_2 X_3 + a_{123} X_1 X_2 X_3 + \varepsilon$$

Effect of “blocks”

- From the previous slide it is easy to see that the carrying out experiments in order from 1 do 8 will probably cause “block” effects since factor 3 has four consecutive + and – signs for experiments 1 – 4 and 5 – 8, respectively;
- To avoid it the sequence should be random, for instance 1, 7, 3, 4, 8, 6, 5, 2;
- This procedure eliminates unwanted bias in assessing the impact of a given factor on the output function.

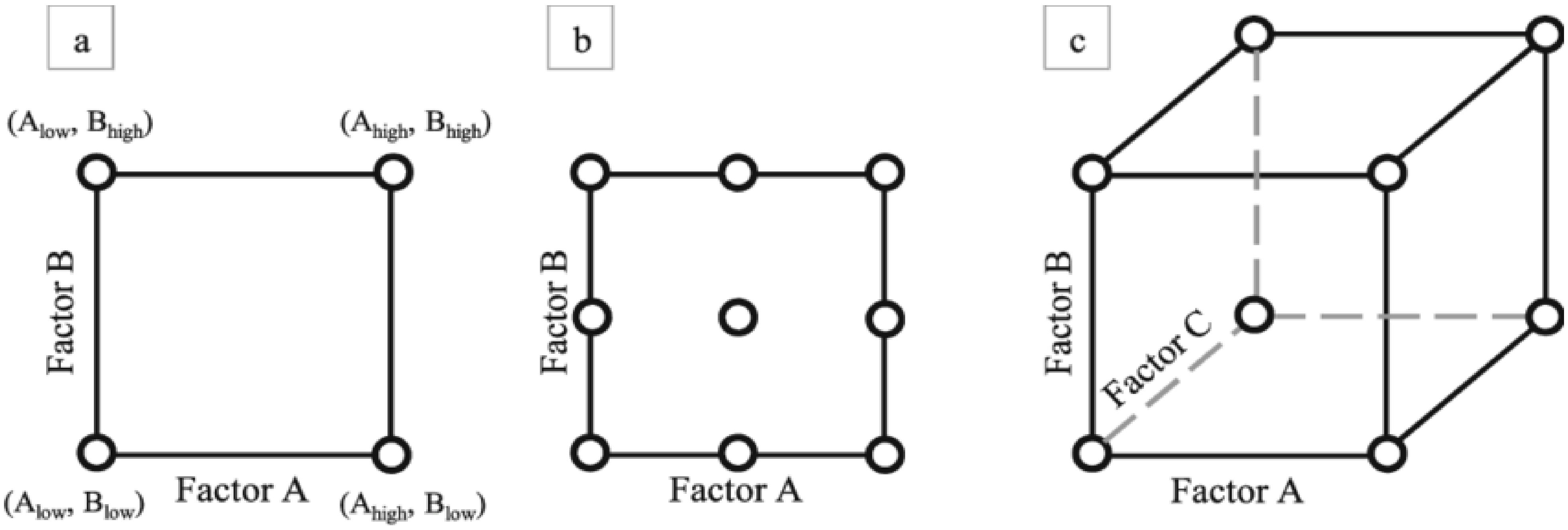
Graphical presentation for FFD

	factor levels	
trial	A	B
1	+	–
2	+	+
3	–	–
4	–	+



	factor levels		
trial	A	B	C
1	+	–	–
2	+	–	+
3	+	+	+
4	+	+	–
5	–	–	–
6	–	–	+
7	–	+	+
8	–	+	–

Another graphical presentation with a “zero” level experiment



Model validation

- In order to assess the quality of the model obtained by the experimental design several assessment steps are necessary:

- Checking the homogeneity of variance;

- Determination of the regression coefficients significance;

- Checking of the adequateness of the model obtained.

Check of experimental variance homogeneity

- It has to be statistically proven that in the experimental work only random errors appear and no systematic deviations are at hand;
- G – test is used: the maximal variance of the series of experiments /sum of all variances gets the experimental values of the test;
- G_{exp} is compared to the G_{theor} (from statistical tables);
- If $G_{exp} < G_{theor}$ the zero-hypothesis is accepted and it means – NO SYSTEMATIC ERROR;
- If systematic error is proven, the whole experimental scheme should be repeated in order to eliminate the problem

Coefficients significance test

- The weight of each input factor is proven by its regression coefficient value and sign;
- If, however, the value of the coefficient is statistically equal to the value of the experimental error, no factor impact on the output could be interpreted – **NO EFFECT JUST EXPERIMENTAL ERROR;**

$$s_{n1}^2 = \frac{\sum s_i^2}{n} \qquad s_{a_i}^2 = \frac{s_{n1}^2}{N}, \qquad N = n \times бр.повторения$$

$$l = \sqrt{s_{a_i}^2 \times t} \quad - ниво\ на\ значимост,\ t_{tabl}(\alpha, N-1)$$

$$|a_i| > l \quad - значим\ регресионен\ коефициент$$

What is the regression coefficient is lower that the merit of significance

- **Each regression coefficient lower that the figure of significance should be eliminated from the final model;**
- **In general, the model indicates the significance (weight) of each single input factor on the output function as well as of each complex combination of factors on the same function; thus, one distinguishes a linear part of the model and a non –linear section;**
- **The lack of coefficient significance means that the respective factor does not affect the output and it leads to reduction of the number of coefficients (respective effects) in the final model.**

After reduction

- If the expected model is of the type given below: FFD 2^3, total number of experiments = 8, total number of regression coefficients 8 (the error estimate is not a regression coefficient, just indication for necessity of error estimation); intercept, three single factor impacts, three double combination of factor and one triple combination, after testing the coefficient significance, the model could be drastically changed:

$$Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_{12}X_1X_2 + a_{13}X_1X_3 + a_{23}X_2X_3 + a_{123}X_1X_2X_3 + \varepsilon$$

Check of the model adequateness

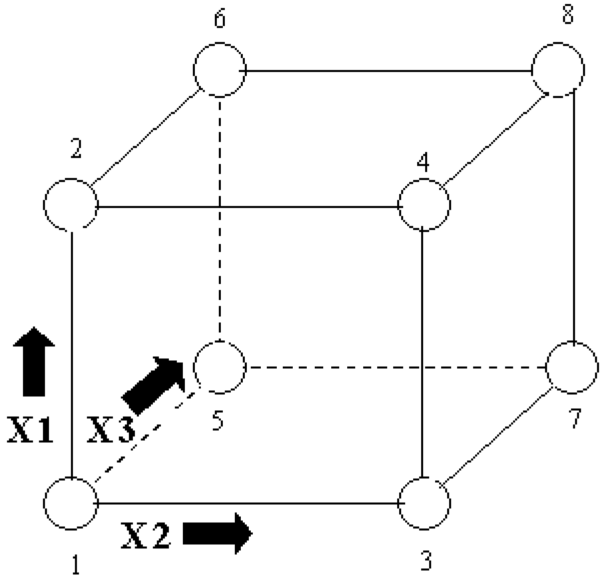
- It has to be checked if the model obtained (calculated) corresponds to the experimental values:
- Least square approach
- Correlation coefficient
- In the first case the checking is based on the application of F – test which compared the variances of the experimental procedure and the variances of the differences between calculated by the model and experimentally obtained results
- The calculation of r or r^2 (calculated vs. experimental results) shows if the model is valid or not

The final model from the same design

- It could seem so:
- $Y = a_0 + a_1X_1 + a_2X_2 + a_3X_3 + a_{123}X_1X_2X_3$
- The double interaction effects proved to be non significant;
- Only linear and third order interactions are important for the system in consideration;
- When checking the model validity only this type of the model will be used for comparison between calculated and experimental values

The applicability of the model obtained

- Each model could be reliably applied within the boundaries of the a priory chosen intervals of variation of the input factors;



Within the cube…

- The model could be applied to each point within the volume of the cube – for 2^3 full factorial design; the experiments are carried out at the corners of the cube but, if adequate. The model can be used for any point from the cube volume;
- There is no guarantee that the model will be adequate outside of this space. It has to be checked by additional experiments or designs with broader intervals of factors change;
- More complex designs could be also applied for more complex models.

Some preliminary conclusions up to now

- Full factorial design depends on factors and levels;
- The most easily applied form of full factorial design is the design on two levels;
- There are variation of the full factorial design – it could be enlarged (e.g. full factorial design on three levels) or reduced (fractional factorial design);
- Full factorial design is not recommendable for more than 5 input factors;
- This limitation could be overcome by applied fractional design.

Economic experimental designs

- In order to reduce the number of experiments necessary for performing factorial designs several economic designs are usually7 applied:
- Fractional factorial design;
- Random balance design;
- Plackett – Burman design