

EXAMPLES FOR THE WZ-METHOD

(FROM “GENERATINGFUNCTIONOLOGY” BY HERBERT WILF)

Theorem. $\sum_k (-1)^k \binom{n}{k} / \binom{k+a}{k} = a/(n+a) \quad (n \geq 0)$

Proof: Take $R(n, k) = k/(n+a)$.

Theorem. $\sum_k (-1)^{n-k} \binom{2n}{k}^2 = \binom{2n}{n} \quad (n \geq 0)$

Proof: Take $R(n, k) = -(10n^2 - 6kn + 17n + k^2 - 5k + 7)/(2(2n - k + 2)^2)$.

Theorem. (*Dixon's identity*)

$$\sum_k (-1)^k \binom{n+b}{n+k} \binom{n+c}{c+k} \binom{b+c}{b+k} = \frac{(n+b+c)!}{n!b!c!}.$$

Proof: Take $R(n, k) = (c+1-k)(b+1-k)/(2(n+k)(n+b+c+1))$.