ЕЗИЦИ, АВТОМАТИ И ИЗЧИСЛИМОСТ
ДИСКРЕТНИ СТРУКТУРИ 2

Задачи върху основните алгоритми

Съдържание

Глава 1

ЗАДАЧИ

1. Крайни Автомати

1.1. Крайни детерминирани автомати.

Задача 1.1. *Докажете, че езикът L е автоматен, където*

(1) $L = \{a^n b | n \geq 0\}$;
(2) $L = \{a^n b | n \geq 1\}$;
(3) $L = \{a^n b^m | n, m \geq 0\}$;
(4) $L = \{a^n b^m | n, m \geq 1\}$;
(5) $L = \{a^n b^m c^k | n, m, k \geq 0\}$;
(6) $L = \{w \in \{0,1\}^* | w$ *започва с* $01\}$;
(7) $L = \{w \in \{0,1\}^* | w$ *завършва с* $01\}$;
(8) $L = \{w \in \{0,1\}^* | w$ *съдържа* $101\}$;
(9) $L = \{w \in \{0,1\}^* | w$ *съдържа точно една* $0\}$;
(10) $L = \{w \in \{0,1\}^* | lh(w) \equiv 0(mod 3)\}$;
(11) $L = \{w \in \{0,1\}^* | N_0(w) \equiv 0(mod 3)\}$;
(12) $L = \{w \in \{0,1\}^* | N_0(w) \equiv 0(mod 3) \& N_1(w) \equiv 1(mod 2)\}$;
(13) $L = \{w \in \{0,1\}^* | w$ *започва и завършва с една и съща буква*$\}$;
(14) $L = \{w \in \{0,1\}^* | w$ *съдържа поне две* $0-$*ли, разделени от непразна дума с дължина кратна на* $4\}$;
(15) $L = \{w \in \{0,1\}^* |$ *всеки път, когато в* w *се появи* 0*, то тя е последвана от поне две* $1\}$;
(16) $L = \{w \in \{0,1\}^* | w$ *съдържа най-много една поддума* $00\}$;
(17) $L = \{w \in \{0,1\}^* |$ *в* w *всяка двойка от съседно нули се среща преди всяка двойка от съседни единици*$\}$;
(18) $L = \{w \in \{a,b,c\}^* \| w$ *не съдържа последователни* b *или* $c\}$;
(19) $L = \{w \in \{0,1\}^* | w$ *е число в двоична бройна система и* $w \equiv 1(mod 3)\}$;
(20) $L = \{w \in \{a,b\}^* | w$ *съдържа като поддума* $bb\}$;
(21) $L = \{w \in \{0,1\}^* | w$ *съдържа нечетен брой* $1\}$;
(22) $L = \{w \in \{0,1\}^* | w$ *съдържа нечетен брой* 1 *и четен брой* $0\}$.

1.2. Тотални автомати. Допълнение на автоматен език.

Задача 1.2. *Намерете краен детерминиран тотален автомат, еквивалентен на автомата:*

(α)

δ	a	b	c
$\to s$	p	s	$-$
p	r	$-$	p
$^*q$	q	r	$-$
$^*r$	r	$-$	$-$

(β)

δ	a	b	c
$\to s$	r	$-$	s
$^*p$	p	s	$-$
$^*q$	p	q	$-$
r	$-$	r	$-$

(γ)

δ	a	b	c
$\to s$	s	p	$-$
$^*p$	q	$-$	p
q	$-$	s	r
$^*r$	$-$	r	r

(δ)

δ	a	b	c
$\to s$	s	$-$	q
p	p	q	p
$^*q$	s	$-$	r
$^*r$	$-$	p	r

(ε)

δ	a	b	c
$\to s$	s	p	$-$
$^*p$	$-$	q	p
$^*q$	s	$-$	r
r	$-$	r	r

(στ)

δ	a	b	c
$\to s$	q	s	s
$^*p$	r	p	$-$
q	$-$	$-$	p
r	s	q	$-$

(ζ)

δ	a	b	c
$\to s$	s	$-$	r
$^*p$	$-$	r	p
q	s	$-$	r
$^*r$	q	p	$-$

(η)

δ	a	b	c
$\to s$	p	q	$-$
p	p	$-$	p
$^*q$	s	$-$	r
$^*r$	$-$	p	$-$

(θ)

δ	a	b	c
$\to s$	s	$-$	p
$^*p$	q	$-$	p
q	$-$	s	r
$^*r$	$-$	r	$-$

(ι)

δ	a	b	c
$\to s$	s	$-$	q
p	p	q	p
$^*q$	s	$-$	r
$^*r$	$-$	p	r

(ια)

δ	a	b	c
$\to s$	p	q	$-$
p	$-$	p	q
$^*q$	q	$-$	$-$

ЗАДАЧА 1.3. *Намерете КДА $\mathcal{A}_1$ със свойството $L(\mathcal{A}_1) = \{a,b\}^* \setminus L(\mathcal{A})$, където $\mathcal{A}$ е автомата:*

(α)

δ	a	b
$\to^* s$	p	r
$^*p$	r	p
$^*q$	s	q
$^*t$	r	q

(β)

δ	a	b
$\to s$	s	p
$^*p$	t	q
$^*q$	t	p
t	s	p

(γ)

δ	a	b
$\to s$	p	q
$^*p$	q	s
q	p	q
$^*t$	q	t

(δ)

δ	a	b
$\to s$	p	q
$^*p$	t	s
$^*q$	q	t
$^*t$	q	p

(ε)

δ	a	b
$\to s$	t	p
$^*p$	q	t
$^*q$	s	t
t	t	s

(στ)

δ	a	b
$\to s$	p	q
p	q	t
$^*q$	t	q
$^*t$	q	s

(ζ)

δ	a	b
$\to^* s$	s	q
p	t	s
q	q	t
$^*t$	q	p

(η)

δ	a	b
$\to s$	s	q
$^*p$	q	s
q	t	p
$^*t$	q	t

(θ)

δ	a	b
$\to s$	p	q
$^*p$	t	s
$^*q$	q	t
$^*t$	q	p

(ι)

δ	a	b
$\to s$	p	q
$^*p$	q	s
q	p	q
$^*t$	q	t

(ια)

δ	a	b
$\to^* s$	t	p
p	q	s
q	p	t
$^*t$	q	s

1.3. Недетерминирани крайни автомати. Детерминизация.

ЗАДАЧА 1.4. *Намерете краен **детерминиран** автомат A_D, еквивалентен на автомата:*

(α)

Δ	a	b
$\to s$	$\{s\}$	$\{s,p\}$
p	$\{q\}$	$\emptyset$
q	$\{r\}$	$\{r\}$
$^*r$	$\{r\}$	$\{r\}$

(β)

Δ	a	b
$\to s$	$\{s,p\}$	$\{s,p\}$
p	$\{q\}$	$\emptyset$
q	$\{r\}$	$\emptyset$
$^*r$	$\{r\}$	$\{r\}$

(γ)

Δ	a	b
$\to s$	$\{s,q\}$	$\{s,p\}$
p	$\emptyset$	$\{q\}$
q	$\emptyset$	$\{r\}$
$^*r$	$\emptyset$	$\{r\}$

(δ)

Δ	a	b
$\to s$	$\{s,p\}$	$\{s,q\}$
p	$\{q\}$	$\emptyset$
q	$\{r\}$	$\emptyset$
$^*r$	$\{r\}$	$\emptyset$

(ε)

Δ	a	b
$\to s$	$\{s\}$	$\{p,q\}$
p	$\{s\}$	$\{r\}$
q	$\{r\}$	$\{s\}$
$^*r$	$\{r\}$	$\emptyset$

(στ)

Δ	a	b
$\to s$	$\{p,q\}$	$\{s\}$
p	$\{r\}$	$\{s\}$
q	$\{s\}$	$\{r\}$
$^*r$	$\emptyset$	$\{r\}$

(ζ)

Δ	a	b	c
$\to p$	$\emptyset$	$\{p\}$	$\{q,r\}$
$^*q$	$\{q,r,s\}$	$\emptyset$	$\{s\}$
r	$\{p,s\}$	$\{q\}$	$\emptyset$
$^*s$	$\{r\}$	$\emptyset$	$\emptyset$

(η)

Δ	a	b	c
$\to p$	$\{p,r\}$	$\{q\}$	$\{q,s\}$
$^*q$	$\emptyset$	$\emptyset$	$\{q\}$
$^*r$	$\{p,s\}$	$\{r,q\}$	$\emptyset$
s	$\{r\}$	$\emptyset$	$\{p,r\}$

1.4. Операции върху автоматни езици.

Задача 1.5. *Намерете* ***недетерминиран*** *автомат C с $L(C) = L(A) \cup L(B)$, където автоматите A и B са:*

(α) A:

Δ	a	b
$\to t$	$\{s,p\}$	$\{p\}$
s	$\{s,p\}$	$\{p\}$
p	$\{p,r\}$	$\{s\}$
$^*r$	$\{r\}$	$\{s,p,r\}$

B:

Δ	a	b
$\to^* v$	$\{q\}$	$\{q,u\}$
$^*q$	$\{q\}$	$\{q,u\}$
$^*u$	$\{q\}$	$\{u\}$

(β) A:

Δ	a	b
$\to^* t$	$\{s,r\}$	$\{p\}$
$^*s$	$\{s,r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s\}$

B:

Δ	a	b
$\to^* v$	$\{q,u\}$	$\{u\}$
q	$\{q,u\}$	$\{u\}$
u	$\{u\}$	$\{q,u\}$

(γ) A:

Δ	a	b
$\to t$	$\{s,r\}$	$\{p\}$
$^*s$	$\{s,r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s\}$

B:

Δ	a	b
$\to^* v$	$\{q,u\}$	$\{u\}$
$^*q$	$\{q,u\}$	$\{q\}$
u	$\{u\}$	$\{q,u\}$

(δ) A:

Δ	a	b
$\to^* v$	$\{s,r\}$	$\{p,r\}$
$^*s$	$\{r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p\}$	$\{s,r\}$

B:

Δ	a	b
$\to w$	$\{q,u\}$	$\{u\}$
$^*q$	$\{q,u\}$	$\{q\}$
u	$\{q\}$	$\{u\}$

(ε) A:

Δ	a	b
$\to^* t$	$\{s,r\}$	$\{p\}$
$^*s$	$\{s,r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s\}$

B:

Δ	a	b
$\to^* v$	$\{q,u\}$	$\{u\}$
$^*q$	$\{u\}$	$\{q\}$
u	$\emptyset$	$\{q,u\}$

($\sigma\tau$) A:

Δ	a	b
$\to^* v$	$\{s,r\}$	$\{p,r\}$
s	$\{r\}$	$\{p,s\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p\}$	$\{s,r\}$

B:

Δ	a	b
$\to w$	$\{q,u\}$	$\{u\}$
q	$\{q\}$	$\{q\}$
$^*u$	$\{q,u\}$	$\{u\}$

(ζ) A:

δ	a	b
$\to s$	r	p
p	r	p
q	r	$-$
$^*r$	$-$	p

B:

δ	a	b
$\to^* 0$	1	2
^{*1}	1	2
2	1	1

(η) A :

δ	a	b
$\to s$	$-$	r
p	$-$	r
q	r	$-$
$^*r$	p	q

B :

δ	a	b
$\to^* 0$	2	1
^{*1}	2	1
2	1	1

Задача 1.6. *Намерете* ***недетерминиран*** *автомат* C *с* $L(C) = L(A) \circ L(B)$.

(α) A :

Δ	a	b
$\to t$	$\{s,p\}$	$\{s\}$
s	$\{s,p\}$	$\{s\}$
$^*p$	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,r\}$	$\emptyset$

B :

Δ	a	b
$\to^* v$	$\{u\}$	$\emptyset$
$^*q$	$\{u\}$	$\emptyset$
u	$\emptyset$	$\{q\}$

(β) A :

Δ	a	b
$\to t$	$\{s\}$	$\{s,r\}$
s	$\{s\}$	$\{s,r\}$
$^*p$	$\{p\}$	$\{s,p\}$
$^*r$	$\{p,r\}$	$\emptyset$

B :

Δ	a	b
$\to^* v$	$\emptyset$	$\{u\}$
$^*q$	$\emptyset$	$\{u\}$
u	$\{u\}$	$\emptyset$

(γ) A :

Δ	a	b
$\to^* v$	$\{u\}$	$\emptyset$
$^*q$	$\{u\}$	$\emptyset$
u	$\emptyset$	$\{q\}$

B :

Δ	a	b
$\to^* t$	$\{p\}$	$\{s,p\}$
s	$\{s,p\}$	$\{s\}$
$^*p$	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,r\}$	$\emptyset$

(δ) A :

Δ	a	b
$\to t$	$\{s,p\}$	$\{s\}$
s	$\{s,p\}$	$\{s\}$
$^*p$	$\{p,r\}$	$\{p\}$
$^*r$	$\{s,r\}$	$\{s,p\}$

B :

Δ	a	b
$\to^* v$	$\{u\}$	$\emptyset$
q	$\{u\}$	$\emptyset$
$^*u$	$\emptyset$	$\{q\}$

(ε) A :

Δ	a	b
$\to^* v$	$\{u\}$	$\emptyset$
$^*q$	$\{u\}$	$\{u,q\}$
u	$\emptyset$	$\{q\}$

B :

Δ	a	b
$\to t$	$\{p\}$	$\{s,p\}$
s	$\{s\}$	$\{s,r\}$
$^*p$	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,r\}$	$\emptyset$

$(\sigma\tau)$ A :

Δ	a	b
$\to t$	$\{s,p\}$	$\{s\}$
s	$\{s\}$	$\{s,p\}$
$^*p$	$\{p,r\}$	$\{r\}$
$^*r$	$\{s,r\}$	$\{s,p\}$

B :

Δ	a	b
$\to^* v$	$\{u\}$	$\emptyset$
q	$\{u\}$	$\{q,u\}$
$^*u$	$\emptyset$	$\{q\}$

(ζ) A :

δ	a	b
$\to s$	$-$	r
p	$-$	r
q	r	$-$
$^*r$	p	q

B :

δ	a	b
$\to^* 0$	2	1
^{*1}	2	1
2	1	1

(η) A :

δ	a	b
$\to s$	r	p
p	r	p
q	r	$-$
$^*r$	$-$	p

B :

δ	a	b
$\to^* 0$	1	2
^{*1}	1	2
2	1	1

Задача 1.7. *Намерете* ***недетерминиран*** *автомат* C *с* $L(C) = L^*(A)$, *където* A *е автоматът*

(α)

Δ	0	1
$\to^* t$	$\{s\}$	$\{s,p\}$
$^*s$	$\{s\}$	$\{s,p\}$
$^*p$	$\{r\}$	$\emptyset$
r	$\emptyset$	$\{s,r\}$

(β)

Δ	0	1
$\to^* t$	$\{s,p\}$	$\{s\}$
$^*s$	$\{s,p\}$	$\{s\}$
p	$\{p\}$	$\{s,r\}$
$^*r$	$\{s,p\}$	$\emptyset$

(γ)

Δ	0	1
$\to t$	$\{p\}$	$\{s\}$
$^*s$	$\{s,p\}$	$\{s,r\}$
p	$\{p\}$	$\{r\}$
$^*r$	$\emptyset$	$\{s,p\}$

(δ)

Δ	0	1
$\to t$	$\{s\}$	$\{s,p\}$
$^*s$	$\{s,r\}$	$\{s,p\}$
p	$\{s\}$	$\{p,r\}$
$^*r$	$\{s,p\}$	$\emptyset$

(ε)

Δ	0	1
$\to t$	$\{p,r\}$	$\{s,p\}$
$^*s$	$\{s,p\}$	$\{s,r\}$
p	$\{p\}$	$\{r\}$
$^*r$	$\{s\}$	$\{p\}$

$(\sigma\tau)$

Δ	0	1
$\to t$	$\{s,r\}$	$\{s,p\}$
$^*s$	$\{s\}$	$\{s,p\}$
p	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,p\}$	$\emptyset$

(ζ)

δ	a	b
$\to s$	r	p
p	r	p
q	r	$-$
$^*r$	$-$	p

(η)

δ	a	b
$\to s$	$-$	r
p	$-$	r
q	r	$-$
$^*r$	p	q

(θ)

δ	a	b	c
$\to p$	$\emptyset$	$\{p\}$	$\{q,r\}$
$^*q$	$\{q,r,s\}$	$\emptyset$	$\{s\}$
r	$\{p,s\}$	$\{q\}$	$\emptyset$
$^*s$	$\{r\}$	$\emptyset$	$\emptyset$

Задача 1.8. *Намерете краен* ***детерминиран*** *автомат* C, *за който* $L(C) = L(A) \cap L(B)$, *където автоматите* A *и* B *са:*

(α) A :

δ	a	b
$\to p$	p	q
q	$-$	r
$^*r$	p	$-$

B :

δ	a	b
$\to^* q$	q	r
r	q	q

(β) A :

δ	a	b
$\to p$	q	p
$^*q$	r	$-$
r	$-$	p

B :

δ	a	b
$\to q$	q	r
$^*r$	q	q

(γ) A :

δ	a	b
$\to p$	r	p
q	r	$-$
$^*r$	$-$	p

B :

δ	a	b
$\to^* q$	q	r
r	q	q

(δ) A :

δ	a	b
$\to p$	r	$-$
q	$-$	r
$^*r$	q	p

B :

δ	a	b
$\to^* q$	q	r
r	q	q

(ε) A :

δ	a	b
$\to^* p$	p	r
q	r	$-$
$^*r$	$-$	p

B :

δ	a	b
$\to^* q$	q	r
r	q	q

$(\sigma\tau)$ A :

δ	a	b
$\to^* p$	r	$-$
q	q	r
$^*r$	$-$	p

B :

δ	a	b
$\to^* q$	q	r
r	q	q

(ζ) A :

δ	a	b
$\to s$	$-$	r
p	$-$	r
q	r	$-$
$^*r$	p	q

B :

δ	a	b
$\to^* 0$	2	1
^{*1}	2	1
2	1	1

(η) A :

δ	a	b
$\to s$	r	p
p	r	p
q	r	$-$
$^*r$	$-$	p

B :

δ	a	b
$\to^* 0$	1	2
^{*1}	1	2
2	1	1

1.5. Минимизация.

Задача 1.9. *Използвайте общ алгоритъм, за да минимизирате автомата:*

(α)

δ	0	1
$\to s$	p	q
p	s	r
$^*q$	t	p
$^*r$	t	s
t	r	s

(β)

δ	0	1
$\to s$	p	q
p	s	r
$^*q$	t	p
$^*r$	t	s
$^*t$	r	s

(γ)

δ	0	1
$\to s$	r	q
$^*p$	w	s
q	p	s
$^*r$	w	q
$^*t$	p	w
w	q	r

(δ)

δ	0	1
$\to s$	r	q
$^*p$	t	s
q	t	s
$^*r$	w	q
$^*t$	w	s
w	q	p

(ε)

δ	0	1
$\to s$	s	p
p	t	s
q	s	r
r	u	q
$^*t$	u	p
$^*u$	t	r

($\sigma\tau$)

δ	0	1
$\to s$	p	s
p	s	t
q	r	s
r	q	u
$^*t$	p	u
$^*u$	r	t

(ζ)

δ	0	1
$\to s$	p	s
p	s	t
q	r	s
r	q	u
$^*t$	p	u
$^*u$	r	t

(η)

δ	0	1
$\to s$	s	p
p	t	s
q	s	r
r	u	q
$^*t$	u	p
$^*u$	t	r

(θ)

δ	0	1
$\to s$	p	s
p	q	r
q	p	t
$^*r$	t	s
$^*t$	r	s

(ι)

δ	a	b
$\to p$	u	t
q	p	r
r	p	u
$^*s$	r	t
$^*t$	u	t
u	p	q

2. Контекстно Свободни Граматики

2.1. Затвореност относно регулярни операции.

Задача 2.1. *Използвайте обща конструкция, за да построите к.св.г. G с език $L(G) = L(G_1) \cup L(G_2)$, където:*

(1) $G_1 = \langle \{a,b\}, \{S_1\}, S_1, \{S_1 \to \varepsilon | aS_1S_1b\} \rangle$,
$G_2 = \langle \{a,b\}, \{S_2,T_2\}, S_2, \{S_2 \to aT_2T_2 | aS_2b, T_2 \to \varepsilon | bS_2b\} \rangle$;

(2) $G_1 = \langle \{a,b\}, \{S_1\}, S_1, \{S_1 \to \varepsilon | bbS_1 | aS_1b\} \rangle$,
$G_2 = \langle \{a,b\}, \{S_2,T_2\}, S_2, \{S_2 \to T_2bS_2 | aS_2b, T_2 \to \varepsilon | aS_2a\} \rangle$;

(3) $G_1 = \langle \{a,b\}, \{S_1\}, S_1, \{S_1 \to \varepsilon | aS_1S_1b\} \rangle$,
$G_2 = \langle \{a,b\}, \{S_2,T_2\}, S_2, \{S_2 \to T_2bS_2 | aS_2b, T_2 \to \varepsilon | aS_2a\} \rangle$;

(4) $G_1 = \langle \{a,b\}, \{S_1\}, S_1, \{S_1 \to \varepsilon | bbS_1 | aS_1b\} \rangle$,
$G_2 = \langle \{a,b\}, \{S_2,T_2\}, S_2, \{S_2 \to aT_2T_2 | aS_2b, T_2 \to \varepsilon | bS_2b\} \rangle$;

(5) $G_1 = \langle \{S_1\}, \{a,b\}, S_1, \{S_1 \to \varepsilon | S_1aS_1b\} \rangle$,
$G_2 = \langle \{S_2,T_2\}, \{a,b\}, S_2, \{S_2 \to aT_2bS_2 | aS_2b, T_2 \to \varepsilon | aT_2a\} \rangle$;

(6) $G_1 = \langle \{S_1\}, \{a,b\}, S_1, \{S_1 \to \varepsilon | bbS_1aa | aS_1b\} \rangle$,
$G_2 = \langle \{S_2,T_2\}, \{a,b\}, S_2, \{S_2 \to aT_2bT_2 | aS_2b, T_2 \to \varepsilon | T_2bS_2b\} \rangle$;

(7) $G_1 = \langle \{a,b\}, \{S_1\}, S_1, \{S_1 \to \varepsilon | aS_1S_1b | S_1ab\} \rangle$,
$G_2 = \langle \{a,b\}, \{S_2,T_2\}, S_2, \{S_2 \to aT_2T_2 | aS_2bb, T_2 \to \varepsilon | bS_2b\} \rangle$;

(8) $G_1 = \langle \{a,b\}, \{S_1\}, S_1, \{S_1 \to \varepsilon | bS_1b | aS_1S_1b\} \rangle$,
$G_2 = \langle \{a,b\}, \{S_2,T_2\}, S_2, \{S_2 \to T_2bS_2 | aS_2b | ab, T_2 \to \varepsilon | aS_2a\} \rangle$;

(9) $G_1 = \langle\{S_1\}, \{a, b\}, S_1, \{S_1 \to \varepsilon | aaS_1baS_1ab\}\rangle$,
$G_2 = \langle\{S_2, T_2\}, \{a, b\}, S_2, \{S_2 \to aaT_2bbbS_2ab | bS_2a, T_2 \to \varepsilon | bT_2b\}\rangle$;
(10) $G_1 = \langle\{S_1\}, \{a, b, c\}, S_1, \{S_1 \to \varepsilon | bS_1acb | aaS_1b\}\rangle$,
$G_2 = \langle\{S_2, T_2\}, \{a, b, c\}, S_2, \{S_2 \to bT_2T_2a | bS_2ac, T_2 \to \varepsilon | bS_2T_2b\}\rangle$;
(11) $G_1 = \langle\{a, b\}, \{A, B, S\}, A, \{A \to \varepsilon | BbS, S \to BB | AaS, B \to a | aBb\}\rangle$
$G_2 = \langle\{a, b\}, \{C, D, S\}, C, \{C \to ab | aCD, D \to SSb, S \to CSD | bb\}\rangle$

Задача 2.2. *Използвайте обща конструкция, за да построите к.св.г.* G *с език* $L(G) = L(G_1) \circ L(G_2)$, *където:*

(1) $G_1 = \langle\{a, b\}, \{S_1, T_1\}, S_1, \{S_1 \to T_1aS_1, T_1 \to T_1bS_1 | aT_1b | a | \varepsilon\}\rangle$,
$G_2 = \langle\{a, b\}, \{S_2\}, S_2, \{S_2 \to a | aS_2b | bbS_2a | S_2aS_2\}\rangle$;
(2) $G_1 = \langle\{a, b\}, \{S_1, T_1\}, S_1, \{S_1 \to T_1bS_1b, T_1 \to T_1S_1 | bS_1aa | a | \varepsilon\}\rangle$,
$G_2 = \langle\{a, b\}, \{S_2\}, S_2, \{S_2 \to a | aS_2b | S_2aS_2bS_2\}\rangle$;
(3) $G_1 = \langle\{a, b\}, \{S_1, T_1\}, S_1, \{S_1 \to T_1aS_1, T_1 \to T_1bS_1 | aT_1b | a | \varepsilon\}\rangle$,
$G_2 = \langle\{a, b\}, \{S_2\}, S_2, \{S_2 \to a | aS_2b | S_2aS_2bS_2\}\rangle$;
(4) $G_1 = \langle\{a, b\}, \{S_1, T_1\}, S_1, \{S_1 \to T_1bS_1b, T_1 \to T_1S_1 | bS_1aa | a | \varepsilon\}\rangle$,
$G_2 = \langle\{a, b\}, \{S_2\}, S_2, \{S_2 \to a | aS_2b | bbS_2a | S_2aS_2\}\rangle$;
(5) $G_1 = \langle\{S_1, T_1\}, \{a, b\}, S_1, \{S_1 \to T_1aS_1, T_1 \to bS_1T_1 | aT_1b | a | \varepsilon\}\rangle$
$G_2 = \langle\{S_2\}, \{a, b\}, S_2, \{S_2 \to a | aS_2b | S_2aS_2b\}\rangle$;
(6) $G_1 = \langle\{S_1, T_1\}, \{a, b\}, S_1, \{S_1 \to T_1bS_1b, T_1 \to T_1S_1 | bS_1aT_1a | a | \varepsilon\}\rangle$,
$G_2 = \langle\{S_2\}, \{a, b\}, S_2, \{S_2 \to a | aS_2b | bS_2ab | S_2aS_2S_2\}\rangle$;
(7) $G_1 = \langle\{a, b\}, \{S_1, T_1\}, S_1, \{S_1 \to T_1aS_1 | \varepsilon, T_1 \to bT_1S_1 | aT_1b | a | \varepsilon\}\rangle$,
$G_2 = \langle\{a, b\}, \{S_2\}, S_2, \{S_2 \to a | aS_2b | bS_2a | S_2aaS_2\}\rangle$;
(8) $G_1 = \langle\{a, b\}, \{S_1, T_1\}, S_1, \{S_1 \to T_1b | S_1T_1, T_1 \to T_1S_1 | bS_1aa | a | \varepsilon\}\rangle$,
$G_2 = \langle\{a, b\}, \{S_2\}, S_2, \{S_2 \to a | aS_2b | S_2aS_2bS_2\}\rangle$;
(9) $G_1 = \langle\{S_1\}, \{a, b, c\}, S_1, \{S_1 \to \varepsilon | bS_1cab | aaS_1b\}\rangle$,
$G_2 = \langle\{S_2, T_2\}, \{a, b, c\}, S_2, \{S_2 \to cbT_2T_2a | bS_2aa, T_2 \to \varepsilon | bS_2T_2b\}\rangle$;
(10) $G_1 = \langle\{S_1\}, \{a, b\}, S_1, \{S_1 \to \varepsilon | aaS_1bbS_1ab\}\rangle$,
$G_2 = \langle\{S_2, T_2\}, \{a, b\}, S_2, \{S_2 \to aaT_2bbbS_2ab | bS_2a, T_2 \to \varepsilon | bT_2b\}\rangle$;
(11) $G_1 = \langle\{a, b\}, \{A, B, S\}, B, \{B \to AB | SbS, S \to AA | aSa | a, A \to a | aBSb\}\rangle$,
$G_2 = \langle\{a, b\}, \{C, D, S\}, S, \{S \to ab | aCD, D \to CbC, C \to DD | ba\}\rangle$;

Задача 2.3. *Използвайте обща конструкция, за да построите к.св.г.* G *с език* $L(G) = (L(G_1))^*$, *където:*

(1) $G_1 = \langle\{a, b\}, \{S_1\}, S_1, \{S_1 \to b | aS_1b | S_1S_1\}\rangle$;
(2) $G_1 = \langle\{a, b\}, \{S_1\}, S_1, \{S_1 \to b | S_1S_1 | S_1aS_1aS_1\}\rangle$;
(3) $G_1 = \langle\{a, b\}, \{S_1\}, S_1, \{S_1 \to b | aaS_1 | S_1S_1\}\rangle$;
(4) $G_1 = \langle\{a, b\}, \{S_1\}, S_1, \{S_1 \to a | S_1S_1 | aS_1bS_1a\}\rangle$;
(5) $G_1 = \langle\{S_1\}, \{a, b\}, S_1, \{S_1 \to b | aS_1a | S_1bbS_1\}\rangle$;
(6) $G_1 = \langle\{S_1\}, \{a, b\}, S_1, \{S_1 \to a | S_1bS_1 | aS_1bS_1a\}\rangle$;
(7) $G_1 = \langle\{a, b\}, \{S_1\}, S_1, \{S_1 \to b | aS_1b | S_1abS_1\}\rangle$;
(8) $G_1 = \langle\{a, b\}, \{S_1\}, S_1, \{S_1 \to ba | S_1bbS_1 | S_1aS_1aS_1\}\rangle$;
(9) $G_1 = \langle\{S_1\}, \{a, b\}, S_1, \{S_1 \to bb | aS_1S_1S_1 | bS_1S_1\}\rangle$;
(10) $G_1 = \langle\{S_1\}, \{a, b\}, S_1, \{S_1 \to aba | S_1S_1b | aS_1S_1S_1\}\rangle$;
(11) $G_1 = \langle\{a, b\}, \{A, B, S\}, A, \{B \to AB | bSSb | \varepsilon, S \to AaA | aBb | a, A \to b | aBSb\}\rangle$;

2.2. Регулярните езици като КСЕ.

Задача 2.4. *Използвайте обща конструкция, за да построите к.св.г.* G *с език* $L(G) = L(A)$, *където* A *е автоматът:*

(α)

δ	a	b
$\to^* s$	p	q
p	p	r
$^*q$	q	s
r	q	r

(β)

δ	a	b
$\to s$	r	s
$^*p$	p	q
q	q	s
$^*r$	s	q

(γ)

δ	a	b
$\to^* s$	r	s
p	s	r
$^*q$	s	q
r	q	s

(δ)

δ	a	b
$\to s$	q	s
$^*p$	s	r
q	s	p
$^*r$	r	q

(ε)

δ	a	b
$\to^* s$	r	s
p	s	q
$^*q$	r	q
r	q	s

($\sigma\tau$)

δ	a	b
$\to s$	q	s
$^*p$	r	s
q	q	p
$^*r$	s	q

(ζ)

δ	a	b
$\to^* s$	p	q
p	p	s
q	r	p
$^*r$	q	r

(η)

δ	a	b
$\to s$	q	s
p	r	q
$^*q$	q	p
$^*r$	p	s

(θ)

δ	a	b
$\to^* 0$	1	0
1	3	0
^{*2}	0	3
^{*3}	1	2

(ι)

δ	a	b
$\to 0$	1	0
^{*1}	1	2
^{*2}	3	0
^{*3}	0	1

($\iota\alpha$)

δ	a	b
$\to^* s$	$\{p, q\}$	$\emptyset$
$\to p$	$\{r, s\}$	$\{p\}$
q	$\{s, q\}$	$\{p, s\}$
$^*r$	$\{r, s\}$	$\{r, p\}$

2.3. Нормална форма на Чомски.

Задача 2.5. *Използвайте обща конструкция, за да намерите всички нетерминали, от които в* Γ *се извежда празната дума. Принадлежи ли празната дума на езика на* Γ*? Обосновете се.*

(1) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S, \{S \to D, D \to AD|b, A \to ACB|BC|a, B \to ABCA|CEC, C \to \varepsilon|CA|a, E \to \varepsilon|aEb\}\rangle$;
(2) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S, \{S \to aD, D \to \varepsilon|ABBA|ADD, A \to DEB|a, B \to DDD|DC|b, C \to CCE|a, E \to \varepsilon|bEa\}\rangle$;
(3) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S, \{S \to D, D \to AD|b, A \to AB|BC|a, B \to AB|CC, C \to \varepsilon|CA|a, E \to a|EB\}\rangle$;
(4) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S, \{S \to AD|a, D \to \varepsilon|BB|AD, A \to DB|a, B \to DD|DC|b, C \to CE|a, E \to AB|b|EA\}\rangle$;
(5) $\Gamma = \langle\{a, b\}, \{S, A, B, C\}, S, \{S \to AS|SB|SS, B \to CA|b, C \to AA|a|BA, A \to \varepsilon|BS\}\rangle$;
(6) $\Gamma = \langle\{a, b\}, \{S, A, B, C\}, S, \{S \to AB|AC, B \to \varepsilon|BC|b, A \to BB|CC|a, C \to CS|a\}\rangle$;
(7) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S, \{S \to DD, D \to AD|b, A \to BC|a, B \to AB|CC, C \to \varepsilon|AC|a, E \to a|EB\}\rangle$;
(8) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S, \{S \to DA|a, D \to \varepsilon|BD|AB, A \to DD|a, B \to CC|DC|b, C \to EC|a, E \to BA|b|AE\}\rangle$;

Задача 2.6. *Нека множеството от терминали в* Γ*, което извежда празната дума е* $\mathcal{E}$*. Използвайте обща конструкция, за да намерите граматика* Γ_1 *без* ε*-правила, за която* $L(\Gamma_1) = L(\Gamma) \setminus \{\varepsilon\}$*. Принадлежи ли празната дума на* $L(\Gamma)$*? Обосновете се!*

(1) $\Gamma = \langle\{a, b\}, \{S, A, B, C\}, S, \{S \to AS|SB|SS, B \to AC|b, C \to A|a|AB, A \to \varepsilon|BS\}\rangle$, $\mathcal{E} = \{A, B, C\}$;
(2) $\Gamma = \langle\{a, b\}, \{S, A, B, C\}, S, \{S \to BA|CA, B \to \varepsilon|BC|b, A \to BB|CC|a, C \to CS|a\}\rangle$, $\mathcal{E} = \{A, B, S\}$;
(3) $\Gamma = \langle\{a, b\}, \{S, A, B, C\}, S, \{S \to AS|b, A \to AC|BC|a, B \to BC|CC, C \to \varepsilon|CA|a\}\rangle$, $\mathcal{E} = \{A, B, C\}$;

(4) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to \varepsilon|BA|AS, A \to SB|a, B \to SS|SC|b, C \to CC|a\}\rangle$, $\mathcal{E} = \{A,B,S\}$;
(5) $\Gamma = \langle\{0,1\},\{S,A,B\},S,\{S \to SB|AS|SS, A \to \varepsilon|0|1, B \to AA|SC, C \to AB|1\}\rangle$, $\mathcal{E} = \{A,B,C\}$;

Задача 2.7. *Използвайте обща конструкция, за да премахнете "дългите"правила (т.е. с дължина поне 2, които не са в н.ф. на Чомски) от* G_1 *като при това получите к.св. граматика* G *с език* $L(G) = L(G_1)$, *където:*

(1) $G_1 = \langle\{S,T\},\{a,b\},S,\{S \to \epsilon|ab|aTba, T \to aTTb\}\rangle$;
(2) $G_1 = \langle\{S,T\},\{a,b\},S,\{S \to \epsilon|ab|baTb, T \to TaTb\}\rangle$;
(3) $\Gamma = \langle\{a,b\},\{A,B,C,S\},S,\{A \to BSB|a, B \to ba|BC, C \to BaSA|a|b, S \to CC|b\}\rangle$;
(4) $\Gamma = \langle\{a,b\},\{A,B,C,S\},S,\{A \to BAS, B \to CB, C \to ab|ABbS, S \to CC|b\}\rangle$;
(5) $G_1 = \langle\{S,T\},\{a,b\},S,\{S \to \epsilon|ab|aTba, T \to TabTT|a\}\rangle$;
(6) $G_1 = \langle\{S,T\},\{a,b\},S,\{S \to \epsilon|ab|bTba, T \to aTaTb|b\}\rangle$;

Задача 2.8. *Използвайте обща конструкция, за да премахнете преименуващите правила от граматиката* Γ *като при това запазите езика.*

(1) $\Gamma = \langle\{a,b\},\{A,B,C,S\},S,\{A \to B|S, B \to C|BC, C \to AB|a|b, S \to B|CC|b\}\rangle$;
(2) $\Gamma = \langle\{a,b\},\{A,B,C,S\},S,\{A \to B, B \to S|C|BC, C \to a|AB, S \to C|CC|b\}\rangle$;
(3) $\Gamma = \langle\{a,b\},\{A,B,C,S\},S,\{A \to B|CC|a, B \to S|AB, C \to SC|b, S \to A|CC|b\}\rangle$;
(4) $\Gamma = \langle\{a,b\},\{A,B,C,S\},S,\{A \to BB|b, B \to S|SS|b, C \to B|a, S \to C|AB|a\}\rangle$;
(5) $\Gamma = \langle\{a,b\},\{A,B,S,C,E\},A,\{A \to BS|a, S \to BB|A, B \to b|CB|A, E \to B|S|AC, C \to E|a|AS\}\rangle$;

Задача 2.9. *Построите к. св. г.* G *в нормална форма на Чомски с език* $L(G) = L(G_1)$, *където:*

(1) $G_1 = \langle\{S,T\},\{a,b\},S,\{S \to \epsilon|ab|aTba, T \to aTTb\}\rangle$;
(2) $G_1 = \langle\{S,T\},\{a,b\},S,\{S \to \epsilon|ab|baTb, T \to TaTb\}\rangle$;

2.4. CYK алгоритъм.

Задача 2.10. *Използвайте алгоритъма за динамично програмиране (CYK), за да проверите дали думата* α *принадлежи на езика, определен от граматиката* Γ.

(1) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to a|AB|AC, C \to SB|AS, A \to a, B \to b\}\rangle$, $\alpha = aaabb$;
(2) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to BA|CA|a, C \to BS|SA, A \to a, B \to b\}\rangle$, $\alpha = bbaaa$;
(3) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to AB|BC, A \to BA|a, B \to CC|b, C \to AB|a\}\rangle$, $\alpha = baaba$;
(4) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to AB, A \to AC|a|b, B \to CB|a, C \to a\}\rangle$, $\alpha = babaa$;
(5) $\Gamma = \langle\{S,A,B\},\{a,b\},S,\{S \to BA|SS|b, A \to SA|a, B \to BS|b\}\rangle$, $bbbaa$;

(6) $\Gamma = \langle\{S, A, B\}, \{a, b\}, S, \{S \to AB|SS|a, A \to AS|a, B \to SB|b\}\rangle$, $aaabb$;
(7) $\Gamma = \langle\{a, b\}, \{S, A, B\}, S, \{S \to AB|BS|b, A \to SS|a, B \to BA|b\}\rangle$, $\alpha = babab$;
(8) $\Gamma = \langle\{a, b\}, \{S, A, B\}, S, \{S \to BA|AS|a, A \to AB|a, B \to SS|b\}\rangle$, $\alpha = ababa$;
(9) $\Gamma = \langle\{a, b\}, \{S, A, B\}, S, \{S \to AB|a, A \to BA|SS|a, B \to SS|b\}\rangle$, $\alpha = aabba$;
(10) $\Gamma = \langle\{a, b\}, \{A, B, S\}, A, \{A \to BS|a, B \to AS|BB|b, S \to BS|AB|a\}\rangle$, $\alpha = abbab$

2.5. Стекови автомати.

Задача 2.11. *Използвайте обща конструкция, за да построите стеков автомат $\mathcal{A}$, който разпознава с празен стек и такъв, че $L(\mathcal{A}) = L(\Gamma)$, където:*

(1) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S,$
$\{S \to aD, D \to ab|ABBA|ADD, A \to DEB|a, B \to DDD|DC|b, C \to CCE|a, E \to ba|bEa\}\rangle$;
(2) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S,$
$\{S \to D, D \to AD|b, A \to ACB|BC|a, B \to ABCA|CEC, C \to \varepsilon|CA|a, E \to ab|aEb\}\rangle$;
(3) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S,$
$\{S \to aD, D \to \varepsilon|ABBA|ADD, A \to DEB|a, B \to DDD|DC|b, C \to CCE|a, E \to \varepsilon|bEa\}\rangle$;
(4) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S,$
$\{S \to D, D \to AD|b, A \to ACB|BC|a, B \to ABCA|CEC, C \to \varepsilon|CA|a, E \to \varepsilon|aEb\}\rangle$;
(5) $\Gamma = \langle\{a, b\}, \{S, A, B\}, S, \{S \to SA|\varepsilon, A \to BSa|B, B \to b|BS|ab\}\rangle$;
(6) $\Gamma = \langle\{a, b\}, \{S, A, B\}, S, \{S \to AS|\varepsilon, A \to SaBB|A, B \to b|BBbS|AA\}\rangle$;
(7) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S,$
$\{S \to DD, D \to DDA|b, A \to CAB|a, B \to BCA|CCE, C \to \varepsilon|CA|a, E \to \varepsilon|EaE\}\rangle$;
(8) $\Gamma = \langle\{S, A, B, C, D, E\}, \{a, b\}, S,$
$\{S \to DD, D \to DA|b, A \to CAB|a, B \to BCA|CCE, C \to \varepsilon|CA|a, E \to \varepsilon|EaE\}\rangle$;

Глава 2

ОТГОВОРИ

1. Крайни Автомати

1.1. Крайни детерминирани автомати.

Задача 1.1.

1.2. Тотални автомати. Допълнение на автоматен език.

Задача 1.2.

(α)

δ	a	b	c
$\to s$	p	s	t
p	r	t	p
$^*q$	q	r	t
$^*r$	r	t	t
t	t	t	t

(β)

δ	a	b	c
$\to s$	r	t	s
$^*p$	p	s	t
$^*q$	p	q	t
r	t	r	t
t	t	t	t

(γ)

δ	a	b	c
$\to s$	s	p	t
$^*p$	q	t	p
q	t	s	r
$^*r$	t	r	r
t	t	t	t

(δ)

δ	a	b	c
$\to s$	s	t	q
p	p	q	p
$^*q$	s	t	r
$^*r$	t	p	r
t	t	t	t

(ε)

δ	a	b	c
$\to s$	s	p	t
$^*p$	t	q	p
$^*q$	s	t	r
r	t	r	r
t	t	t	t

$(\sigma\tau)$

δ	a	b	c
$\to s$	q	s	s
$^*p$	r	p	t
q	t	t	p
r	s	q	t
t	t	t	t

(ζ)

δ	a	b	c
$\to s$	s	t	r
$^*p$	t	r	p
q	s	t	r
$^*r$	q	p	t
t	t	t	t

(η)

δ	a	b	c
$\to s$	p	q	t
p	p	t	p
$^*q$	s	t	r
$^*r$	t	p	t
t	t	t	t

(θ)

δ	a	b	c
$\to s$	s	t	p
$^*p$	q	t	p
q	t	s	r
$^*r$	t	r	t
t	t	t	t

(ι)

δ	a	b	c
$\to s$	s	t	q
p	p	q	p
$^*q$	s	t	r
$^*r$	t	p	r
t	t	t	t

Задача 1.3.

(α)

δ	a	b
$\to s$	p	r
p	r	p
q	s	q
t	r	q

(β)

δ	a	b
$\to^* s$	s	p
p	t	q
q	t	p
$^*t$	s	p

(γ)

δ	a	b
$\to^* s$	p	q
p	q	s
$^*q$	p	q
t	q	t

(δ)

δ	a	b
$\to^* s$	p	q
p	t	s
q	q	t
t	q	p

(ε)

δ	a	b
$\to^* s$	t	p
p	q	t
q	s	t
$^*t$	t	s

$(\sigma\tau)$

δ	a	b
$\to^* s$	p	q
$^*p$	q	t
q	t	q
t	q	s

(ζ)

δ	a	b
$\to s$	s	q
$^*p$	t	s
$^*q$	q	t
t	q	p

(η)

δ	a	b
$\to^* s$	s	q
p	q	s
$^*q$	t	p
t	q	t

(θ)

δ	a	b
$\to^* s$	p	q
p	t	s
q	q	t
t	q	p

(ι)

δ	a	b
$\to^* s$	p	q
p	q	s
$^*q$	p	q
t	q	t

1.3. Недетерминирани крайни автомати. Детерминизация.

Задача 1.4.

(α)

δ	a	b
$\to s$	s	sp
sp	sq	sp
sq	sr	spr
$^*sr$	sr	spr
$^*spr$	sqr	spr
$^*sqr$	sr	spr

(β)

δ	a	b
$\to s$	sp	sp
sp	spq	sp
spq	$spqr$	sp
$^*spqr$	$spqr$	spr
$^*spr$	$spqr$	spr

(γ)

δ	a	b
$\to s$	sq	sp
sp	sq	spq
sq	sq	spr
$^*spr$	sq	$spqr$
spq	sq	$spqr$
$^*spqr$	sq	$spqr$

(δ)

δ	a	b
$\to s$	sp	sq
sp	spq	sq
sq	spr	sq
spq	$spqr$	sq
$^*spr$	$spqr$	sq
$^*spqr$	$spqr$	sq

(ε)

δ	a	b
$\to s$	s	pq
pq	sr	sr
$^*sr$	sr	pq

($\sigma\tau$)

δ	a	b
$\to s$	pq	s
pq	sr	sr
$^*sr$	pq	sr

(ζ)

δ	a	b	c
$\to \{p\}$	$\emptyset$	$\{p\}$	$\{q,r\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$^*\{q,r\}$	$\{p,q,r,s\}$	$\{q\}$	$\{s\}$
$^*\{p,q,r,s\}$	$\{p,q,r,s\}$	$\{p,q\}$	$\{q,r,s\}$
$^*\{q\}$	$\{q,r,s\}$	$\emptyset$	$\{s\}$
$^*\{s\}$	$\{r\}$	$\emptyset$	$\emptyset$
$^*\{p,q\}$	$\{q,r,s\}$	$\{p\}$	$\{q,r,s\}$
$^*\{q,r,s\}$	$\{p,q,r,s\}$	$\{q\}$	$\{s\}$

(η)

δ	a	b	c
$\to \{p\}$	$\{p,r\}$	$\{q\}$	$\{q,s\}$
$^*\{p,r\}$	$\{p,r,s\}$	$\{q,r\}$	$\{q,s\}$
$^*\{q\}$	$\emptyset$	$\emptyset$	$\{q\}$
$^*\{q,s\}$	$\{r\}$	$\emptyset$	$\{p,q,r\}$
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
$^*\{r\}$	$\{p,s\}$	$\{q,r\}$	$\emptyset$
$^*\{p,q,r\}$	$\{p,r,s\}$	$\{q,r\}$	$\{q,s\}$
$\{p,s\}$	$\{p,r\}$	$\{q,r\}$	$\{q,s\}$
$^*\{q,r\}$	$\{p,s\}$	$\{q,r\}$	$\{q\}$
$^*\{p,r,s\}$	$\{p,r,s\}$	$\{q,r\}$	$\{p,q,r,s\}$
$\{p,q,r,s\}$	$\{p,r,s\}$	$\{q,r\}$	$\{p,q,r,s\}$

1.4. Операции върху автоматни езици.

Задача 1.5.

(α) C :

Δ	a	b
$\to^* n$	$\{s,p,q\}$	$\{p,q,u\}$
t	$\{s,p\}$	$\{p\}$
s	$\{s,p\}$	$\{p\}$
p	$\{p,r\}$	$\{s\}$
$^*r$	$\{r\}$	$\{s,p,r\}$
$^*v$	$\{q\}$	$\{q,u\}$
$^*q$	$\{q\}$	$\{q,u\}$
$^*u$	$\{q\}$	$\{u\}$

(β) C :

Δ	a	b
$\to^* n$	$\{s,r,q,u\}$	$\{p,u\}$
$^*t$	$\{s,r\}$	$\{p\}$
$^*s$	$\{s,r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s\}$
$^*v$	$\{q,u\}$	$\{u\}$
q	$\{q,u\}$	$\{u\}$
u	$\{u\}$	$\{q,u\}$

(γ) C :

Δ	a	b
$\to^* n$	$\{s,r,q,u\}$	$\{p,u\}$
t	$\{s,r\}$	$\{p\}$
$^*s$	$\{s,r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s\}$
$^*v$	$\{q,u\}$	$\{u\}$
$^*q$	$\{q,u\}$	$\{q\}$
u	$\{u\}$	$\{q,u\}$

(δ) C :

Δ	a	b
$\to^* n$	$\{s,r,q,u\}$	$\{p,r,u\}$
$^*v$	$\{s,r\}$	$\{p,r\}$
$^*s$	$\{r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p\}$	$\{s,r\}$
w	$\{q,u\}$	$\{u\}$
$^*q$	$\{q,u\}$	$\{q\}$
u	$\{q\}$	$\{u\}$

(ε) C :

Δ	a	b
$\to^* n$	$\{s,r,q,u\}$	$\{p,u\}$
$^*t$	$\{s,r\}$	$\{p\}$
$^*s$	$\{s,r\}$	$\{p\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s\}$
$^*v$	$\{q,u\}$	$\{u\}$
$^*q$	$\{u\}$	$\{q\}$
u	$\emptyset$	$\{q,u\}$

$(\sigma\tau)$ C :

Δ	a	b
$\to^* n$	$\{s,r,q,u\}$	$\{p,r,u\}$
$^*v$	$\{s,r\}$	$\{p,r\}$
s	$\{r\}$	$\{p,s\}$
p	$\emptyset$	$\{r\}$
$^*r$	$\{s,p\}$	$\{s,r\}$
w	$\{q,u\}$	$\{u\}$
q	$\{q\}$	$\{q\}$
$^*u$	$\{q,u\}$	$\{u\}$

(ζ) C :

Δ	a	b
$\to^* n$	$\{r,1\}$	$\{p,2\}$
s	$\{r\}$	$\{p\}$
p	$\{r\}$	$\{p\}$
q	$\{r\}$	$\emptyset$
$^*r$	$\emptyset$	$\{p\}$
^{*0}	$\{1\}$	$\{2\}$
^{*1}	$\{1\}$	$\{2\}$
2	$\{1\}$	$\{1\}$

(η) C :

Δ	a	b
$\to^* n$	$\{2\}$	$\{r,1\}$
s	$\emptyset$	$\{r\}$
p	$\emptyset$	$\{r\}$
q	$\{r\}$	$\emptyset$
$^*r$	$\{p\}$	$\{q\}$
^{*0}	$\{2\}$	$\{1\}$
^{*1}	$\{2\}$	$\{1\}$
2	$\{1\}$	$\{1\}$

Задача 1.6.

(α) C :

Δ	a	b
$\to t$	$\{s,p\}$	$\{s\}$
s	$\{s,p\}$	$\{s\}$
$^*p$	$\{p,u\}$	$\{p,r\}$
$^*r$	$\{s,r,u\}$	$\emptyset$
$^*v$	$\{u\}$	$\emptyset$
$^*q$	$\{u\}$	$\emptyset$
u	$\emptyset$	$\{q\}$

(β) C :

Δ	a	b
$\to t$	$\{s\}$	$\{s,r\}$
s	$\{s\}$	$\{s,r\}$
$^*p$	$\{p\}$	$\{s,p,u\}$
$^*r$	$\{p,r\}$	$\{u\}$
$^*v$	$\emptyset$	$\{u\}$
$^*q$	$\emptyset$	$\{u\}$
u	$\{u\}$	$\emptyset$

(γ) C :

Δ	a	b
$\to^* v$	$\{u,p\}$	$\{s,p\}$
$^*q$	$\{u,p\}$	$\{s,p\}$
u	$\emptyset$	$\{q\}$
$\to^* t$	$\{p\}$	$\{s,p\}$
s	$\{s,p\}$	$\{s\}$
$^*p$	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,r\}$	$\emptyset$

(δ) C :

Δ	a	b
$\to t$	$\{s,p\}$	$\{s\}$
s	$\{s,p\}$	$\{s\}$
$^*p$	$\{p,r,u\}$	$\{p\}$
$^*r$	$\{s,r,u\}$	$\{s,p\}$
$^*v$	$\{u\}$	$\emptyset$
q	$\{u\}$	$\emptyset$
$^*u$	$\emptyset$	$\{q\}$

(ε) C :

Δ	a	b
$\to v$	$\{u,p\}$	$\{s,p\}$
q	$\{u,p\}$	$\{u,q,s,p\}$
u	$\emptyset$	$\{q\}$
t	$\{p\}$	$\{s,p\}$
s	$\{s\}$	$\{s,r\}$
$^*p$	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,r\}$	$\emptyset$

$(\sigma\tau)$ C :

Δ	a	b
$\to t$	$\{s,p\}$	$\{s\}$
s	$\{s\}$	$\{s,p\}$
$^*p$	$\{p,r,u\}$	$\{r\}$
$^*r$	$\{s,r,u\}$	$\{s,p\}$
$^*v$	$\{u\}$	$\emptyset$
q	$\{u\}$	$\{q,u\}$
$^*u$	$\emptyset$	$\{q\}$

(ζ) C :

δ	a	b
$\to s$	$\emptyset$	$\{r\}$
p	$\emptyset$	$\{r\}$
q	$\{r\}$	$\emptyset$
$^*r$	$\{p,2\}$	$\{q,1\}$
^{*0}	$\{2\}$	$\{1\}$
^{*1}	$\{2\}$	$\{1\}$
2	$\{1\}$	$\{1\}$

(η) C :

δ	a	b
$\to s$	$\{r\}$	$\{p\}$
p	$\{r\}$	$\{p\}$
q	$\{r\}$	$\emptyset$
$^*r$	$\{1\}$	$\{p,2\}$
^{*0}	$\{1\}$	$\{2\}$
^{*1}	$\{1\}$	$\{2\}$
2	$\{1\}$	$\{1\}$

Задача 1.7.

(α)

Δ	0	1
$\to^* t$	$\{s\}$	$\{s,p\}$
$^*s$	$\{s\}$	$\{s,p\}$
$^*p$	$\{r,s\}$	$\{s,p\}$
r	$\emptyset$	$\{s,r\}$

(β)

Δ	0	1
$\to^* t$	$\{s,p\}$	$\{s\}$
$^*s$	$\{s,p\}$	$\{s\}$
p	$\{p\}$	$\{s,r\}$
$^*r$	$\{s,p\}$	$\{s\}$

(γ)

Δ	0	1
$\to^* t$	$\{p\}$	$\{s\}$
$^*s$	$\{s,p\}$	$\{s,r\}$
p	$\{p\}$	$\{r\}$
$^*r$	$\{p\}$	$\{s,p\}$

(δ)

Δ	0	1
$\to^* t$	$\{s\}$	$\{s,p\}$
$^*s$	$\{s,r\}$	$\{s,p\}$
p	$\{s\}$	$\{p,r\}$
$^*r$	$\{s,p\}$	$\{s,p\}$

(ε)

Δ	0	1
$\to^* t$	$\{p,r\}$	$\{s,p\}$
$^*s$	$\{s,p,r\}$	$\{s,r,p\}$
p	$\{p\}$	$\{r\}$
$^*r$	$\{s,p,r\}$	$\{s,p\}$

$(\sigma\tau)$

Δ	0	1
$\to^* t$	$\{s,r\}$	$\{s,p\}$
$^*s$	$\{s,r\}$	$\{s,p\}$
p	$\{p\}$	$\{p,r\}$
$^*r$	$\{s,p,r\}$	$\{s,p\}$

(ζ)

δ	a	b
$\to^* s$	$\{r\}$	$\{p\}$
p	$\{r\}$	$\{p\}$
q	$\{r\}$	$\emptyset$
$^*r$	$\{r\}$	$\{p\}$

(η)

δ	a	b
$\to^* s$	$\emptyset$	$\{r\}$
p	$\emptyset$	$\{r\}$
q	$\{r\}$	$\emptyset$
$^*r$	$\{p\}$	$\{q,r\}$

(θ)

Δ	a	b	c
$\to^* t$	$\emptyset$	$\{p\}$	$\{q,r\}$
p	$\emptyset$	$\{p\}$	$\{q,r\}$
$^*q$	$\{q,r,s\}$	$\{p\}$	$\{q,r,s\}$
r	$\{p,s\}$	$\{q\}$	$\emptyset$
$^*s$	$\{r\}$	$\{p\}$	$\{q,r\}$

Задача 1.8.

(α) C :

δ	a	b
$\to pq$	pq	qr
qr	$-$	rq
$^*rq$	pq	$-$

(β) C :

δ	a	b
$\to p$	$-$	$-$

(γ) C :

δ	a	b
$\to pq$	rq	pr
$^*rq$	$-$	pr
pr	rq	pq

(δ) C :

δ	a	b
$\to pq$	rq	$-$
$^*rq$	qq	pr
qq	$-$	rr
pr	rq	$-$
rr	qq	pq

(ε) C :

δ	a	b
$\to^* pq$	pq	rr
rr	$-$	pq

($\sigma\tau$) C :

δ	a	b
$\to^* pq$	rq	$-$
$^*rq$	$-$	pr
pr	rq	$-$

(ζ) C :

δ	a	b
$\to s0$	$-$	$r1$
$^*r1$	$p2$	$q1$
$p2$	$-$	$r1$
$q1$	$r2$	$-$
$r2$	$p1$	$q1$
$p1$	$-$	$r1$

(η) C :

δ	a	b
$\to s0$	$r1$	$p2$
$^*r1$	$-$	$p2$
$p2$	$r1$	$p1$
$p1$	$r1$	$p2$

1.5. Минимизация.

Задача 1.9.

(α)

δ	0	1
$\to sp$	sp	qr
$^*qr$	t	sp
t	qr	sp

(β)

δ	0	1
$\to sp$	sp	qrt
$^*qrt$	qrt	sp

(γ)

δ	0	1
$\to sq$	pr	sq
$^*pr$	w	sq
$^*t$	pr	w
w	sq	pr

(δ)

δ	0	1
$\to sq$	rt	sq
$^*p$	rt	sq
$^*rt$	w	sq
w	sq	p

(ε)

δ	0	1
$\to sq$	sq	pr
pr	tu	sq
$^*tu$	tu	pr

($\sigma\tau$)

δ	0	1
$\to sq$	pr	sq
pr	sq	tu
$^*tu$	pr	tu

(ζ)

δ	0	1
$\to sq$	pr	sq
pr	sq	tu
$^*t$	pr	tu

(η)

δ	0	1
$\to sq$	sq	pr
pr	tu	sq
$^*t$	tu	pr

(ι)

δ	a	b
Q_0	Q_2	Q_0
$^*Q_1$	Q_0	Q_1
$\to Q_2$	Q_0	Q_1

2. Контекстно Свободни Граматики

2.1. Затвореност относно регулярни операции.

Задача 2.1. (1) $G = \langle\{a,b\},\{S_1,S_2,T_2,S\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|aS_1S_1b, S_2 \to aT_2T_2|aS_2b, T_2 \to \varepsilon|bS_2b\}\rangle$;
(2) $G = \langle\{a,b\},\{S_1,S_2,T_2,S\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|bbS_1|aS_1b, S_2 \to T_2bS_2|aS_2b, T_2 \to \varepsilon|aS_2a\}\rangle$;
(3) $G = \langle\{a,b\},\{S_1,S_2,T_2,S\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|aS_1S_1b, S_2 \to T_2bS_2|aS_2b, T_2 \to \varepsilon|aS_2a\}\rangle$;
(4) $G = \langle\{a,b\},\{S_1,S_2,T_2,S\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|bbS_1|aS_1b, S_2 \to aT_2T_2|aS_2b, T_2 \to \varepsilon|bS_2b\}\rangle$;
(5) $G = \langle\{S_1,S_2,T_2,S\},\{a,b\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|S_1aS_1b, S_2 \to aT_2bS_2|aS_2b, T_2 \to \varepsilon|aT_2a\}\rangle$;
(6) $G = \langle\{S_1,S_2,T_2,S\},\{a,b\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|bbS_1aa|aS_1b, S_2 \to aT_2bT_2|aS_2b, T_2 \to \varepsilon|T_2bS_2b\}\rangle$;
(7) $G = \langle\{a,b\},\{S_1,S_2,T_2,S\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|aS_1S_1b|S_1ab, S_2 \to aT_2T_2|aS_2bb, T_2 \to \varepsilon|bS_2b\}\rangle$;
(8) $G = \langle\{a,b\},\{S_1,S_2,T_2,S\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|bS_1b|aS_1S_1b, S_2 \to T_2bS_2|aS_2b|ab, T_2 \to \varepsilon|aS_2a\}\rangle$;
(9) $G = \langle\{S_1,S_2,T_2,S\},\{a,b\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|aaS_1baS_1ab, S_2 \to aaT_2bbbS_2ab|bS_2a, T_2 \to \varepsilon|bT_2b\}\rangle$;
(10) $G = \langle\{S_1,S_2,T_2,S\},\{a,b,c\},S,$
$\{S \to S_1|S_2, S_1 \to \varepsilon|bS_1acb|aaS_1b, S_2 \to bT_2T_2a|bS_2ac, T_2 \to \varepsilon|bS_2T_2b\}\rangle$;
(11) $G = \langle\{a,b\},\{A,B,C,D,S_1,S_2,S\},S,\{S \to A|C, A \to \varepsilon|BbS_1,$
$S_1 \to BB|AaS_1, B \to a|aBbC \to ab|aCD, D \to S_2S_2b, S_2 \to CS_2D|bb\}\rangle$

Задача 2.2. (1) $G = \langle\{a,b\},\{S_1,T_1,S_2,S\},S,$
$\{S \to S_1S_2, S_1 \to T_1aS_1, T_1 \to T_1bS_1|aT_1b|a|\varepsilon, S_2 \to a|aS_2b|bbS_2a|S_2aS_2\}\rangle$;
(2) $G = \langle\{a,b\},\{S_1,T_1,S_2,S\},S,$
$\{S \to S_1S_2, S_1 \to T_1bS_1b, T_1 \to T_1S_1|bS_1aa|a|\varepsilon, S_2 \to a|aS_2b|S_2aS_2bS_2\}\rangle$;
(3) $G = \langle\{a,b\},\{S_1,T_1,S_2,S\},S,$
$\{S \to S_1S_2, S_1 \to T_1aS_1, T_1 \to T_1bS_1|aT_1b|a|\varepsilon, S_2 \to a|aS_2b|S_2aS_2bS_2\}\rangle$;
(4) $G = \langle\{a,b\},\{S_1,T_1,S_2,S\},S,$
$\{S \to S_1S_2, S_1 \to T_1bS_1b, T_1 \to T_1S_1|bS_1aa|a|\varepsilon, S_2 \to a|aS_2b|bbS_2a|S_2aS_2\}\rangle$;
(5) $G = \langle\{S_1,T_1,S_2,S\},\{a,b\},S,$
$\{S \to S_1S_2, S_1 \to T_1aS_1, T_1 \to bS_1T_1|aT_1b|a|\varepsilon, S_2 \to a|aS_2b|S_2aS_2b\}\rangle$;
(6) $G = \langle\{S_1,T_1,S_2,S\},\{a,b\},S,$
$\{S \to S_1S_2, S_1 \to T_1bS_1b, T_1 \to T_1S_1|bS_1aT_1a|a|\varepsilon, S_2 \to a|aS_2b|bS_2ab|S_2aS_2S_2\}\rangle$;
(7) $G = \langle\{a,b\},\{S_1,T_1,S_2,S\},S,$
$\{S \to S_1S_2, S_1 \to T_1aS_1|\varepsilon, T_1 \to bT_1S_1|aT_1b|a|\varepsilon, S_2 \to a|aS_2b|bS_2a|S_2aaS_2\}\rangle$;
(8) $G = \langle\{a,b\},\{S_1,T_1,S_2,S\},S,$
$\{S \to S_1S_2, S_1 \to T_1b|S_1T_1, T_1 \to T_1S_1|bS_1aa|a|\varepsilon, S_2 \to a|aS_2b|S_2aS_2bS_2\}\rangle$;
(9) $G = \langle\{S_1,S_2,T_2,S\},\{a,b,c\},S,$
$\{S \to S_1S_2, S_1 \to \varepsilon|bS_1cab|aaS_1b, S_2 \to cbT_2T_2a|bS_2aa, T_2 \to \varepsilon|bS_2T_2b\}\rangle$;
(10) $G = \langle\{S_1,S_2,T_2,S\},\{a,b\},S,$
$\{S \to S_1S_2, S_1 \to \varepsilon|aaS_1bbS_1ab, S_2 \to aaT_2bbbS_2ab|bS_2a, T_2 \to \varepsilon|bT_2b\}\rangle$;
(11) $G = \langle\{a,b\},\{A,B,C,D,S_1,S_2,S\},S,\{S \to BS_2, B \to AB|S_1bS_1, S_1 \to$
$AA|aS_1a|a, A \to a|aBS_1bS_2 \to ab|aCD, D \to CbC, C \to DD|ba\}\rangle$

Задача 2.3. (1) $G = \langle\{a,b\},\{S_1,S\},S,\{S \to \varepsilon|SS_1, S_1 \to b|aS_1b|S_1S_1\}\rangle$;

(2) $G = \langle\{a,b\},\{S_1,S\},S,\{S \to \varepsilon|SS_1, S_1 \to b|S_1S_1|S_1aS_1aS_1\}\rangle$;
(3) $G = \langle\{a,b\},\{S_1,S\},S,\{S \to \varepsilon|SS_1, S_1 \to b|aaS_1|S_1S_1\}\rangle$;
(4) $G = \langle\{a,b\},\{S_1,S\},S,\{S \to \varepsilon|SS_1, S_1 \to a|S_1S_1|aS_1bS_1a\}\rangle$;
(5) $G = \langle\{S_1,S\},\{a,b\},S,\{S \to \varepsilon|SS_1, S_1 \to b|aS_1a|S_1bbS_1\}\rangle$;
(6) $G = \langle\{S_1,S\},\{a,b\},S,\{S \to \varepsilon|SS_1, S_1 \to a|S_1bS_1|aS_1bS_1a\}\rangle$;
(7) $G = \langle\{a,b\},\{S_1,S\},S,\{S \to \varepsilon|SS_1, S_1 \to b|aS_1b|S_1abS_1\}\rangle$;
(8) $G = \langle\{a,b\},\{S_1,S\},S,\{S \to \varepsilon|SS_1, S_1 \to ba|S_1bbS_1|S_1aS_1aS_1\}\rangle$;
(9) $G = \langle\{S_1,S\},\{a,b\},S,\{S \to \varepsilon|SS_1, S_1 \to bb|aS_1S_1S_1|bS_1S_1\}\rangle$;
(10) $G = \langle\{S_1,S\},\{a,b\},S,\{S \to \varepsilon|SS_1, S_1 \to aba|S_1S_1b|aS_1S_1S_1\}\rangle$;
(11) $G = \langle\{a,b\},\{A,B,S,S_0\},S_0,\{S_0 \to \varepsilon|AS_0, B \to AB|bSSb|\varepsilon, S \to AaA|aBb|a, A \to b|aBSb\}\rangle$;

2.2. Регулярните езици като КСЕ.

Задача 2.4.

(α) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to ap|bq|\varepsilon, p \to ap|br, q \to \varepsilon|aq|bs, r \to aq|br\}\rangle$
(β) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to ar|bs, p \to ap|bq|\varepsilon, q \to aq|bs, r \to as|bq|\varepsilon\}\rangle$
(γ) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to ar|bs|\varepsilon, p \to as|br, q \to \varepsilon|as|bq, r \to aq|bs\}\rangle$
(δ) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to aq|bs, p \to as|br|\varepsilon, q \to as|bp, r \to ar|bq|\varepsilon\}\rangle$
(ε) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to ar|bs|\varepsilon, p \to as|bq, q \to \varepsilon|ar|bq, r \to aq|bs\}\rangle$
($\sigma\tau$) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to aq|bs, p \to ar|bs|\varepsilon, q \to aq|bp, r \to as|bq|\varepsilon\}\rangle$
(ζ) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to ap|bq|\varepsilon, p \to ap|bs, q \to ar|bp, r \to aq|br|\varepsilon\}\rangle$
(η) $G = \langle\{a,b\},\{s,p,q,r\},s,\{s \to aq|bs, p \to ar|bq, q \to \varepsilon|aq|bp, r \to ap|bs|\varepsilon\}\rangle$
(θ) $G = \langle\{a,b\},\{0,1,2,3\},0,\{0 \to a1|b0|\varepsilon, 1 \to a3|b0, 2 \to \varepsilon|a0|b3, 3 \to a1|b2|\varepsilon\}\rangle$
(ι) $G = \langle\{a,b\},\{0,1,2,3\},0,\{0 \to a1|b0, 1 \to a1|b2|\varepsilon, 2 \to a3|b0|\varepsilon, 3 \to a0|b1|\varepsilon\}\rangle$
($\iota\alpha$) $G = \langle\{a,b\},\{p,q,r,s,t\},t,\{t \to s|p, s \to ap|aq|\varepsilon, p \to ar|as|bp, q \to as|aq|bp|bs, r \to ar|as|br|bp|\varepsilon\}\rangle$

2.3. Нормална форма на Хомски. Навсякъде в отговорите на този раздел, които съдържат единствено правилата на граматиката, ще имаме предвид, че началният нетерминален символ е S.

Задача 2.5. (1) $\mathcal{E} = \{A,B,C,E\}$, $\varepsilon \notin L(\Gamma)$;
(2) $\mathcal{E} = \{A,B,D,E\}$, $\varepsilon \notin L(\Gamma)$;
(3) $\mathcal{E} = \{A,B,C\}$, $\varepsilon \notin L(\Gamma)$;
(4) $\mathcal{E} = \{A,B,D,E,S\}$, $\varepsilon \in L(\Gamma)$;
(5) $\mathcal{E} = \{A,B,C\}$, $\varepsilon \notin L(\Gamma)$;
(6) $\mathcal{E} = \{A,B,S\}$, $\varepsilon \in L(\Gamma)$;
(7) $\mathcal{E} = \{A,B,C\}$, $\varepsilon \notin L(\Gamma)$;
(8) $\mathcal{E} = \{A,D,S\}$, $\varepsilon \in L(\Gamma)$;

Задача 2.6. (1) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to AS|SB|SS, B \to AC|b|A|C, C \to A|a|AB|B, A \to BS|S\}\rangle$, $\varepsilon \notin L(\Gamma)$;
(2) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to BA|CA|B|A|C, B \to BC|b|C, A \to BB|CC|a|B, C \to CS|a|C\}\rangle$, $\varepsilon \in L(\Gamma)$;
(3) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to AS|b, A \to AC|BC|a|C|B, B \to BC|CC|C, C \to CA|a|C\}\rangle$, $\varepsilon \notin L(\Gamma)$
(4) $\Gamma = \langle\{a,b\},\{S,A,B,C\},S,\{S \to BA|AS|A|B, A \to SB|a|S|B, B \to SS|SC|b|S|C, C \to CC|a\}\rangle$, $\varepsilon \in L(\Gamma)$

(5) $\Gamma_1 = \langle\{0,1\}, \{S, A, B\}, S, \{S \to SB|AS|SS, A \to \varepsilon|0|1, B \to AA|SC|A|S, C \to AB|A|B|1\}\rangle$, $\varepsilon \notin L(\Gamma)$;

Задача 2.7. (1) $S \to AB|AS_1, S_1 \to TS_2, S_2 \to BA, A \to a, B \to b, T \to AT_1, T_1 \to TT_2, T_2 \to TB$;
(2) $S \to AB|BS_1, S_1 \to AS_2, S_2 \to TB, A \to a, B \to b, T \to TT_1, T_1 \to AT_2, T_2 \to TB$;
(3) $A \to BA_1|a, A_1 \to SB, B \to B'A'|BC, A' \to a, B' \to b, C \to BC_1|a|b, C_1 \to A'C_2, C_2 \to SA, S \to CC|b$;
(4) $A \to BA_1, A_1 \to AS, B \to CB, C \to A'B'|AC_1, C_1 \to BC_2, C_2 \to B'S, S \to CC|b, A' \to a, B' \to b$;
(5) $S \to AB|AS_1, S_1 \to TS_2, S_2 \to BA, B \to b, A \to a, T \to TT_1|a, T_1 \to AT_2, T_2 \to BT_3, T_3 \to TT$;
(6) $S \to AB|BS_1, S_1 \to TS_2, S_2 \to BA, B \to b, A \to a, T \to AT_1|b, T_1 \to TT_2, T_2 \to AT_3, T_3 \to TB$;

Задача 2.8. (1) $\Gamma = \langle\{a,b\}, \{A, B, C, S\}, S, \{A \to BC|AB|a|b|CC, B \to BC|AB|a|b, C \to AB|a|b, S \to CC|b|BC|AB|a\}\rangle$;
(2) $\Gamma = \langle\{a,b\}, \{A, B, C, S\}, S, \{A \to BC|CC|b|a|AB, B \to BC|CC|b|a|AB, C \to a|AB, S \to CC|b|a|AB\}\rangle$;
(3) $\Gamma = \langle\{a,b\}, \{A, B, C, S\}, S, \{C \to SC|b, S \to CC|b|a|SS\}\rangle$;
(4) $\Gamma = \langle\{a,b\}, \{A, B, C, S\}, S, \{A \to SS|b, S \to AS|a|SS|b\}\rangle$;
(5) $\Gamma = \langle\{a,b\}, \{A, B, S, C, E\}, A, \{A \to BS|a, S \to BS|a, B \to BS|a, S \to BB, E \to BB|BS|a, B \to b|CB, E \to b|CB, E \to AC, C \to AC|b|CB|BB|BS|a, C \to AS\}\rangle$;

Задача 2.9. (1) $S \to AB|AS_1, S_1 \to TS_2, S_2 \to BA, B \to b, A \to a, T \to AT_1, T_1 \to TT_2, T_2 \to TB$;
(2) $S \to AB|BS_1, S_1 \to AS_2, S_2 \to TB, A \to a, B \to b, T \to TT_1, T_1 \to AT_2, T_2 \to TB$;

2.4. CYK алгоритъм.

Задача 2.10. (1) $\alpha = aaabb \in L(\Gamma)$;
(2) $\alpha = bbaaa \in L(\Gamma)$;
(3) $\alpha = baaba \in L(\Gamma)$;
(4) $\alpha = babaa \notin L(\Gamma)$;
(5) $\alpha = bbbaa \in L(\Gamma)$;
(6) $\alpha = aaabb \in L(\Gamma)$;
(7) $\alpha = babab \in L(\Gamma)$;
(8) $\alpha = ababa \in L(\Gamma)$;
(9) $\alpha = aabba \in L(\Gamma)$;
(10) $\alpha = abbab \notin L(\Gamma)$