$$(\lambda_x M)N \rightsquigarrow M[x \mapsto N]$$

$$\equiv \lambda_w y$$

$$\equiv \lambda_v y$$

$$(\lambda_y x)[x \mapsto y] \equiv \lambda_z y$$

$$\lambda_x x \approx \lambda_y y \approx \lambda_z z \ldots$$

1. $x[x \rightsquigarrow N] := N$
2. $y[x \rightsquigarrow N] := y$ за $y \not\equiv x$
3. $(M_1 M_2)[x \rightsquigarrow N] := (M_1[x \rightsquigarrow N])(M_2[x \rightsquigarrow N])$
4. $(\lambda_y P)[x \rightsquigarrow N] := \lambda_y(P[x \rightsquigarrow N])$

Лема 1

Нека $M, N \in \Lambda$ и $\boxed{x \notin \mathrm{FV}(M)}$. Тогава $M[x \mapsto N] \equiv M$.

Док-во: Инд. по дълж. на $M[x \mapsto N]$

1) $M \equiv x$ — невъзможно! $x \notin FV(M)$

2) $M \equiv y \not\equiv x$ $\quad M[x \mapsto N] \equiv y \equiv M$ ✓

3) $(M_1 M_2)[x \mapsto N] \stackrel{def}{\equiv} (M_1[x \mapsto N])(M_2[x \mapsto N])$

$x \notin FV(M_1)$?

$FV(M_1) \subseteq FV(M) \stackrel{def\ FV.2}{=} FV(M_1) \cup FV(M_2)$

$FV(M_2) \subseteq$

ИП

$\equiv M_1(M_2[x \mapsto N]) \stackrel{ИП}{\equiv} M_1 M_2$

1. $x[x \rightsquigarrow N] := N$
2. $y[x \rightsquigarrow N] := y$ за $y \not\equiv x$
3. $(M_1 M_2)[x \rightsquigarrow N] := (M_1[x \rightsquigarrow N])(M_2[x \rightsquigarrow N])$
4. $(\lambda_y P)[x \rightsquigarrow N] := \lambda_y(P[x \rightsquigarrow N])$

Лема 1

Нека $M, N \in \Lambda$ *и* $x \notin \mathrm{FV}(M)$. *Тогава* $M[x \mapsto N] \equiv M$.

4) $M \equiv \lambda_y P$

$(\lambda_y P)[x \mapsto N] \overset{\text{def. 4}}{\equiv} \lambda_y P[x \mapsto N] \equiv$

$x \notin FV(P)$?

$x \notin FV(M) \overset{\text{def. 3}}{=} FV(P) \setminus \{y\}$

Можем да считаме, че $x \not\equiv y$

$\overset{\text{ИП}}{\equiv} \lambda_y P$ $\square$

$\lambda_x (x\, z)\, (\lambda_y\, y\, \lambda z)$ $\lambda_u (u\, z)\, (\lambda_v\, v\, u\, z)$

$x, y \mapsto u, v$