ICT in SES

Graphics and geometry

Lesson №7

Graphics and dimensions

Fundamentals of computer graphics

Core ideas

- Visualization with computer tools
- 2D scene is a special case of the 3D scene
- Application of analytical geometry

Includes

- Static images
- Dynamic images (animation)

Etymology

- *Anima* (Latin): *life*, *soul*, *breath*, *gust*
- Many derivative words: *animator*, *animalist*, *reanimation*, *anime*, *animism*, …

Relies on

- Speed of modern computers
- Imperfections of human brain
  - People do not see too fast motion
  - People do not see too slow motion

Dimensions

Three types dimensions

Dimensions of graphical objects

- Three types of dimensions
- Objects always have all of them

Types

- Objective dimension
- Spatial dimension
- Visual dimension

Objective dimension

The dimension of the object itself

- Traditional concept of dimension
- Defines the variety of points in the object

Examples

- Points are 0D
- Lines are 1D
- Squares are 2D

Spatial dimension

Object dimension of the space

- Defines the variety of object positions
- Objective dimension ≤ spatial dimension
  - A square could be placed in 3D space
  - A square could not be placed in 1D space

Examples

- Planimetry is 2D space
- Stereometry is 3D space

Visual dimension

Objective dimension of the graphical primitives

- Defines the variety of object properties
- Not restricted by the other two dimensions
  - Square (2D) could be drawn from little spheres (3D)
  - Square (2D) could be drawn from points (0D)

Examples

- Drawing with points is 0D visual dimension
- Drawing with segments in 1D visual dimension

Co-existing dimensions

What dimension is referred in texts?

- Depends on the context
- Example: “3D point” assumes the spatial dimension

Square hovering in the air has:

- 1D visual dimension
- 2D objective dimension
- 3D spatial dimension

Coordinate systems

Coordinate systems

Purpose

- Define the position of an object
- Cartesian, polar, spherical, cylindrical,…

Coordinate system in Suica

- Only 3D Cartesian
- If another is used, its much be converted to Cartesian
- Note: sometime it is good to use non-Cartesian systems

Cartesian coordinate system

Elements

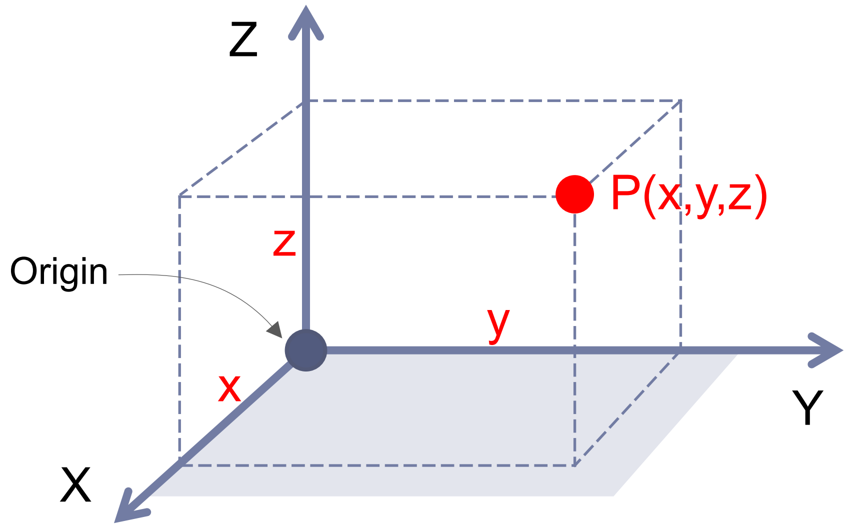
- Origin – a point
- Three orthogonal axes

Axes

- Relative names X, Y, Z
- Equal units
- Relative directions

Coordinates of a point

- Three distances x, y and z, one for each axis
- The origin has coordinates (0,0,0)
- A point on the X axis has coordinates (x,0,0)



2D Cartesian coordinate system

- A special case of the 3D
- If working in XY plane, assume z=0

Transformation

- The Cartesian coordinate system is embeded in Suica
- Coordinates of all other coordinate systems are transformed to 3D Cartesian coordinates

Polar coordinate system

Elements

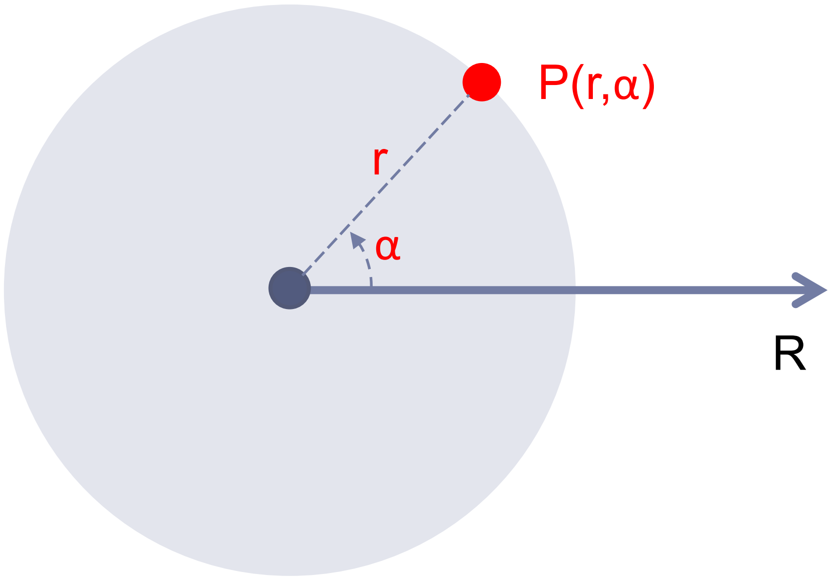
- Origin (pole) – a point
- Polar axis

Axis

- Passes through the pole
- Defines the 0-th direction (angle)
- Angles are relative to the axis

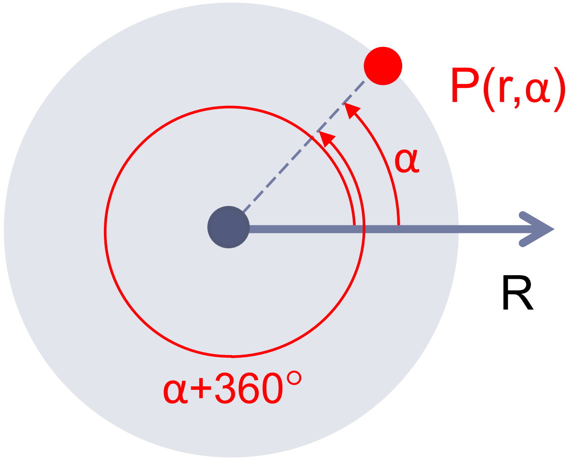
Coordinates of a point

- Distance r to the pole and angle α to the axis
- Coordinates of the origin (0,…)
- Coordinate of a point on the axis (… ,0)



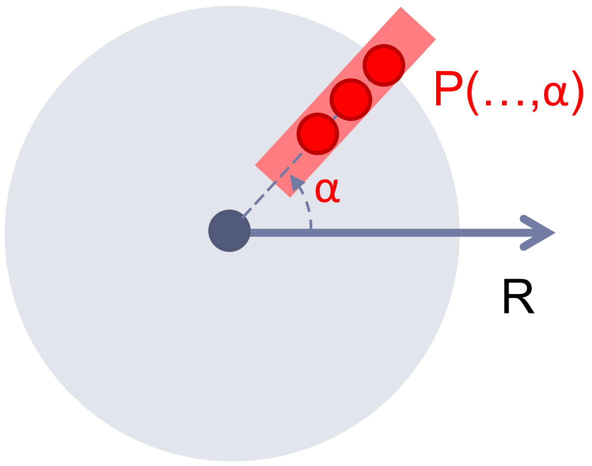
Uniqueness

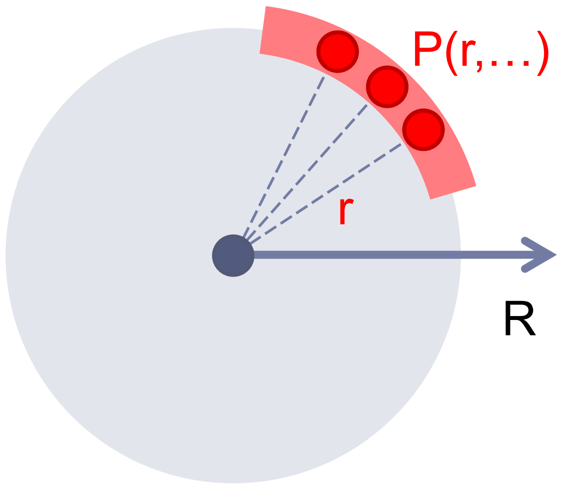
- Each coordinates refer to a single point
- The opposite is not correct, e.g. (r,α)=(r,α+360°)
- The origin has coordinates (0,α) for any α



Coordinate grid

- Points on radial lines have variable radius, fixed angle
- Points on concentric lines have fixed radius, variable angle





Using polar coordinate system

- Rotational motion
- Circular trajectories

Transformation to Cartesian coordinates

- $x = r.\cos(\alpha)$
- $y = r.\sin(\alpha)$

Spherical coordinate system

Elements

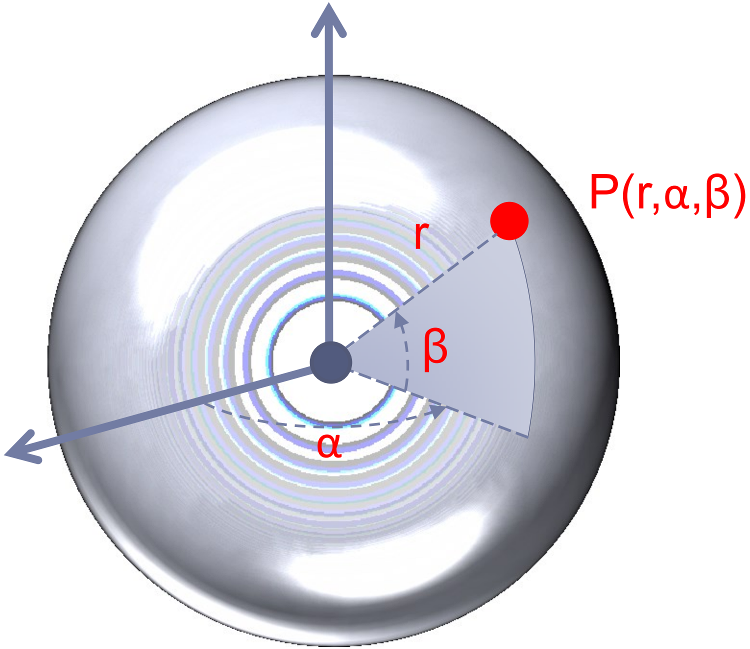
- Origin or pole – a point
- Two orthogonal polar axes

Polar axes

- Pass through the pole
- Define the 0^{th} directions (angles)
- The directions are relative

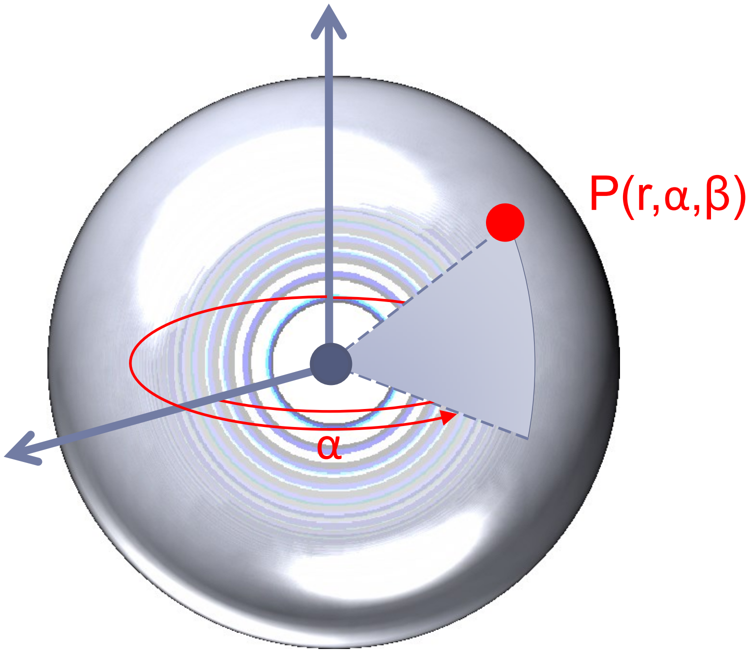
Coordinates of a point

- Distance r to the pole and angles α and β to the axes
- Often β is measured towards the orthogonal plane
- The origin has coordinates (0,... ,...)



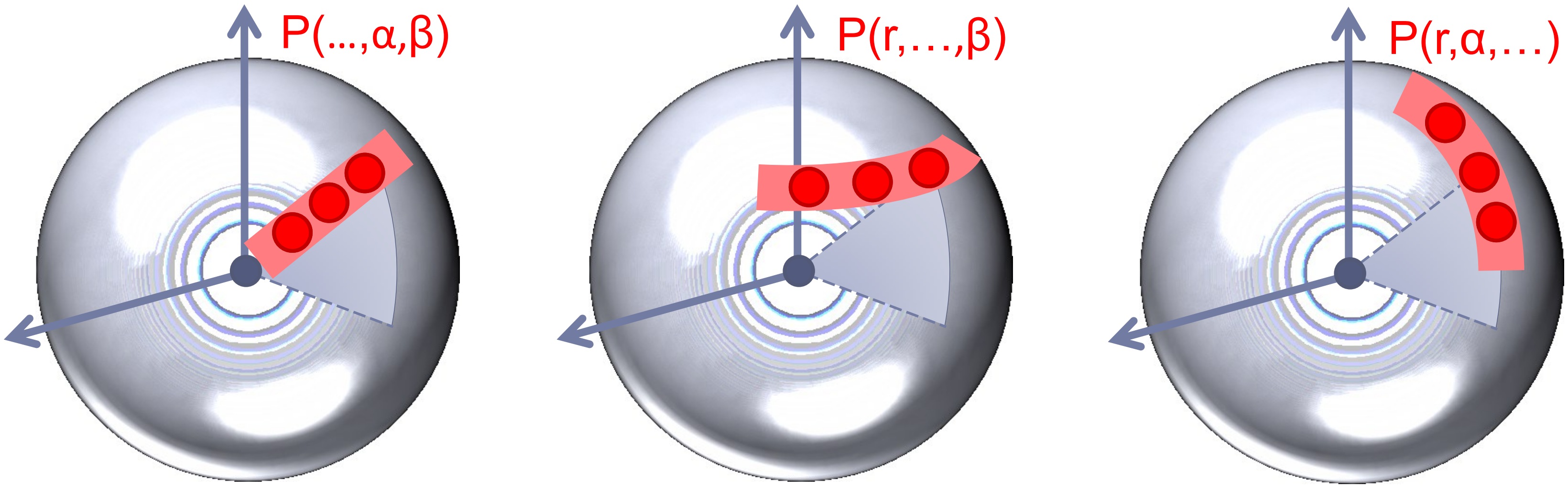
Uniqueness

- Each coordinates refer to a single point
- The opposite is not correct, e.g. $(r,\alpha,\beta)=(r,\alpha+360°,\beta-720°)$
- The origin has coordinates $(0,\alpha,\beta)$ for any α and β



Coordinate grid

- Points on radial lines have variable radius, fixed angles
- Points on concentric lines have fixed radius, fixed one angle and variable another angle



Using polar coordinate system

- Rotational motion in 3D
- Circular or spherical trajectories in 3D

Transformation to Cartesian coordinates

- $x = r.\cos(\alpha).\cos(\beta)$
- $y = r.\sin(\alpha).\cos(\beta)$
- $z = r.\sin(\beta)$
- The polar transformation is spherical transformation with $\beta=0$

Frequently used problems

Manual calculations

Provided functionality

- Suica provides only the basic functionality
- Everything else must be programmed by the user

Frequently used problems

- Translation from one position to another
- Distance between two 3D points
- Finding intermediate point
- Traversing a numerical range

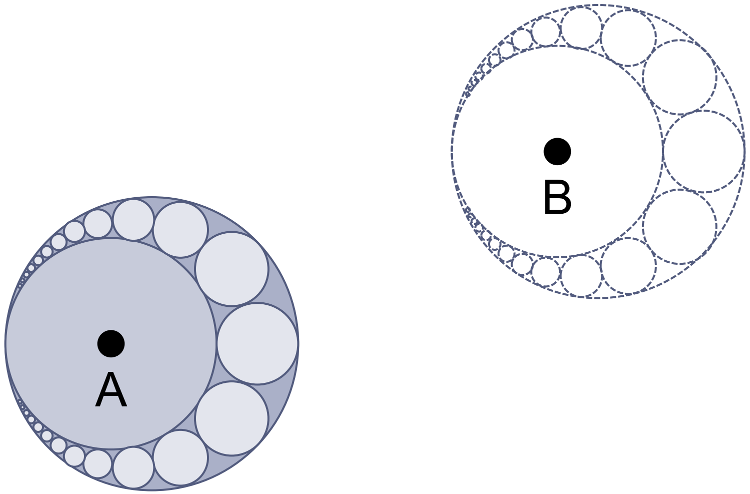
Practically

- Most problems are decomposed to one or more the these four basic problems

Translation

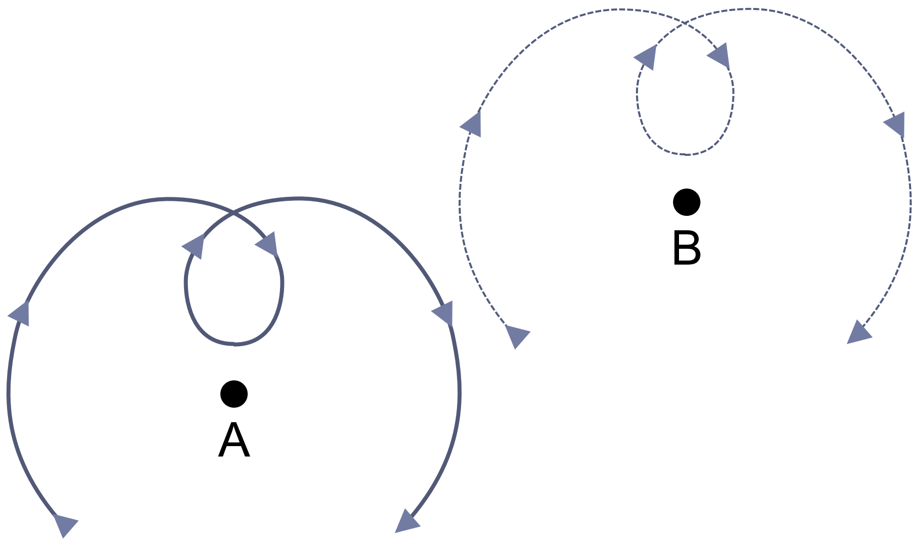
Case №1

- An object exists relative to point A
- The object must be moved to point B



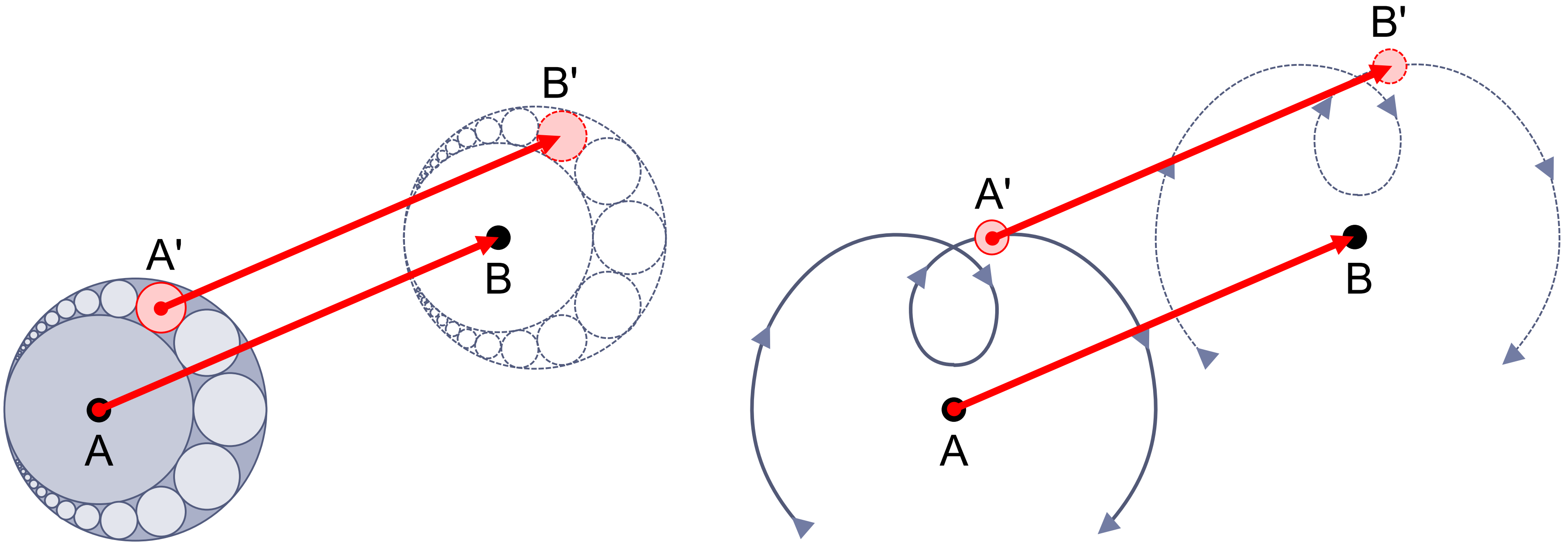
Case №2

- Object flies relative to point A
- The motion must be the same but relative to point B
- It doesn't matter if the trajectory passes through A or B



Solution

- Let points have coordinates $A(x_A,y_A,z_A)$ and $B\ (x_B,y_B,z_B)$
- Point A' is known, B' is unknown
- Using vectors $B' = A' + \overrightarrow{AB}$



Example

Problem

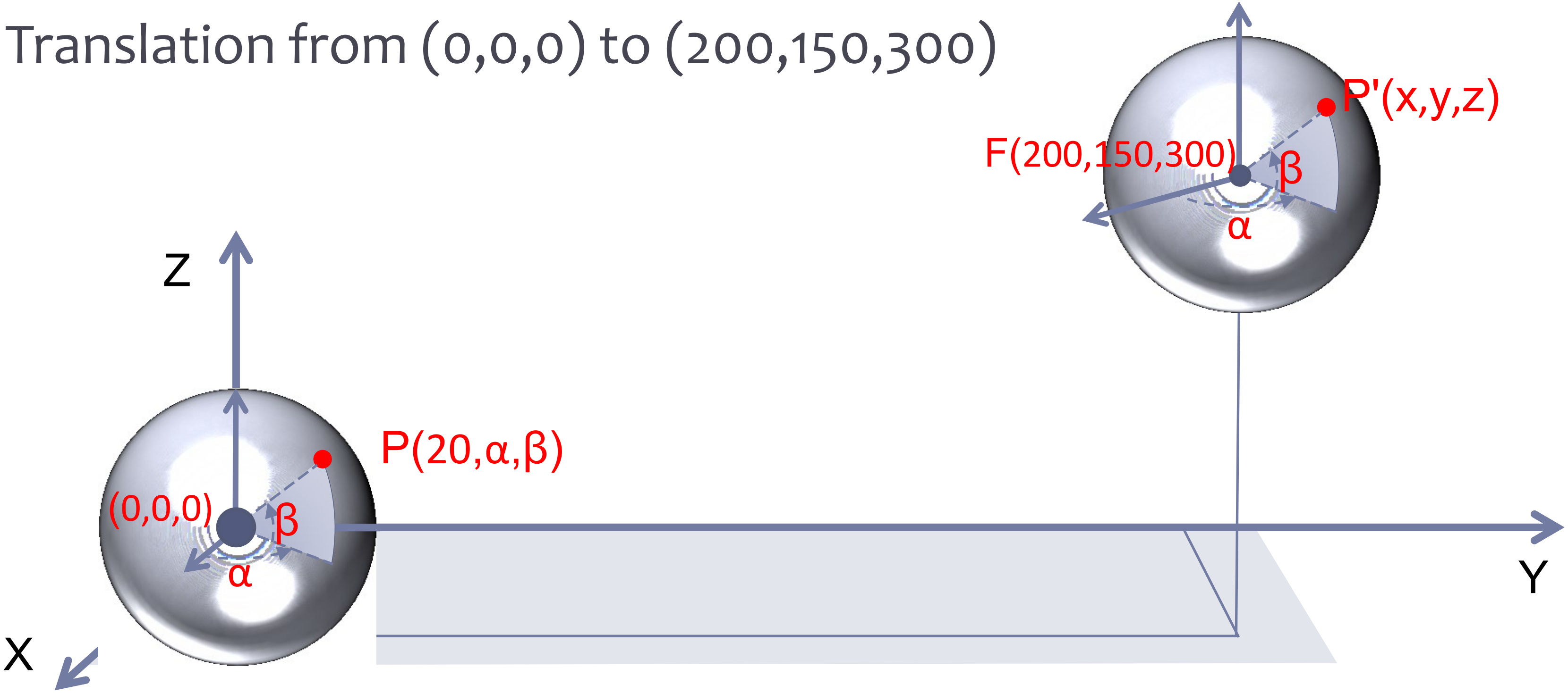
- Point P moves on a sphere with radius 20 and center (0,0,0)
- Spherical coordinates are P(20,α,β), α and β depend on time
- There is point F(200,150,300) in Cartesian coordinates

Goal

- The same motion, but around F, not P

Idea for solution

- P in Cartesian coordinates
- Translation from (0,0,0) to (200,150,300)



Solution

- P in Cartesian coordinates:

$$P_x = 20.\cos(\alpha).\cos(\beta)$$
$$P_y = 20.\sin(\alpha).\cos(\beta)$$
$$P_z = 20.\sin(\beta)$$

- Translation from (0,0,0) to (200,150,300)

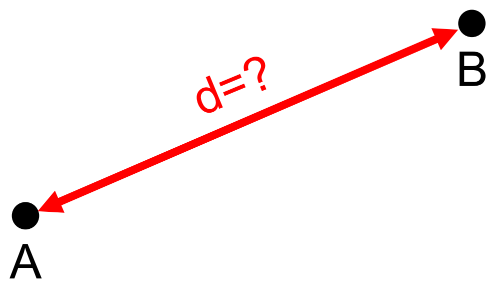
$$P'_x = 200 + 20.\cos(\alpha).\cos(\beta)$$
$$P'_y = 150 + 20.\sin(\alpha).\cos(\beta)$$
$$P'_z = 300 + 20.\sin(\beta)$$

(200-0, 150-0, 300-0)

Distance

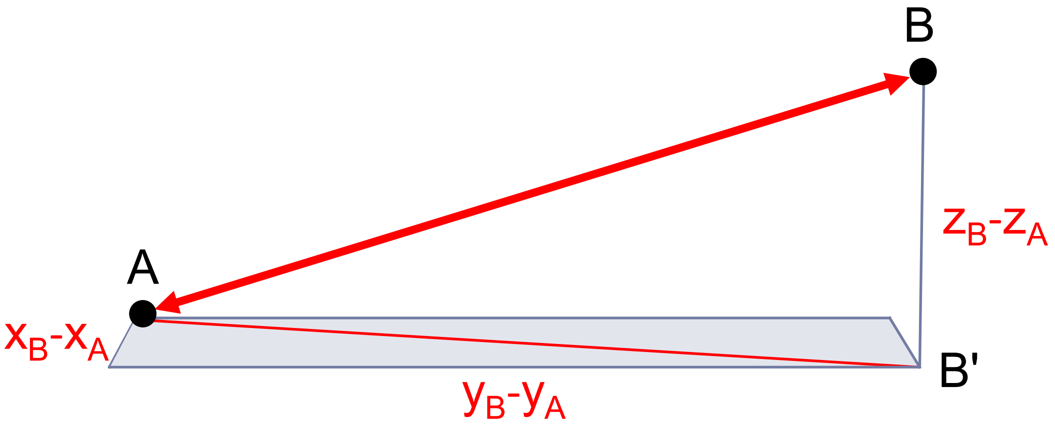
Distance in 3D

- Two 3D points A and B
- What is the distance between them?



Idea for solution

- Pythagoras theorem $|AB'| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$
- Again the theorem $|AB| = \sqrt{|AB'|^2 + (z_B - z_A)^2}$
- Final result $|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}$



Example

- Drawing a tube from (20,2,0) to (10,10,5)
- What is the length of the tube?

Solution

- Direct calculation:

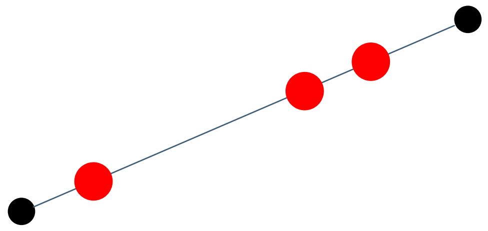
$$\sqrt{(20-10)^2 + (2-10)^2 + (0-5)^2} = \sqrt{100 + 64 + 25} \approx 13.7$$

- Approximately 13.7 meters? Millimeters?

Intermediate position

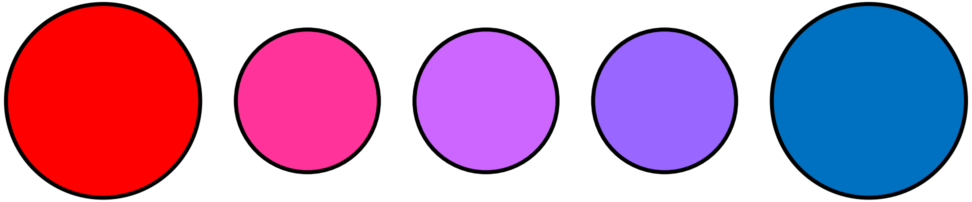
Case №1

- There are two 3D points
- Find points on the line between them



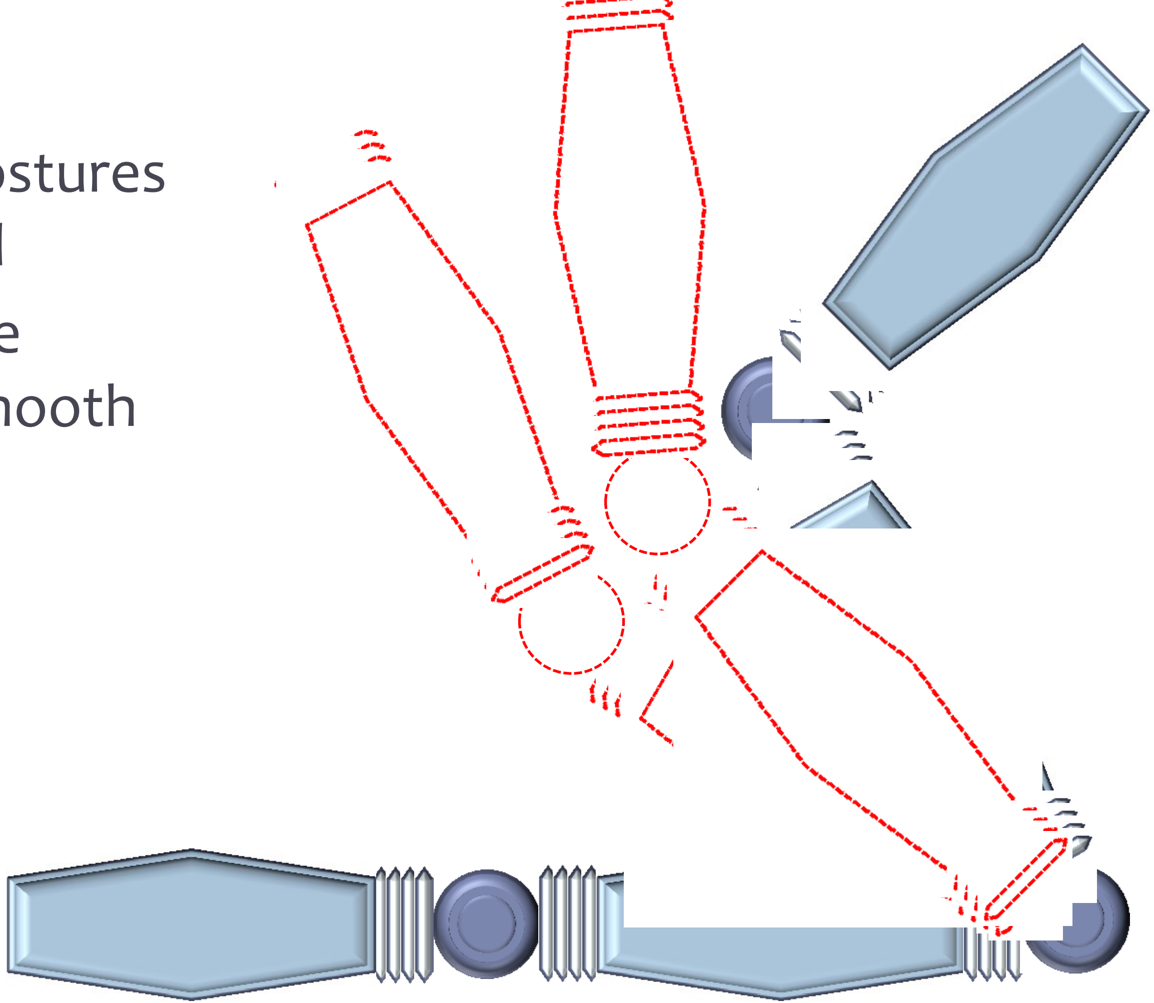
Case №2

- There are two colours
- Find intermediate colours between them



Case №3

- There are two postures of a robotic hand
- Find intermediate postures for a smooth transition



Idea

- Both situations (initial and final) are represented by numbers
- Linear combination between the situations
- There is coefficient, determining the “weight” of each situation
- The middle situation is when the coefficient is 0.5

Step №1: describing with numbers

- Available values must be sufficiently many

  For values [0,1] there is no way to find intermediate numbers

  For values [0.0, 0.1, 0.2, … 0.9, 1.0] it is possible

- Examples

  Coordinates are triplets of numbers (x,y,z)

  Colours are triplets of numbers (r,g,b)

  Positions are pairs of numbers (α,β)

Step №2: choosing coefficient k

- Available values $k \in [0,1]$
- The value of k determined the intermediate position
  - When $k=0$ the position is one of the situations
  - When $k=1$ the position is the other situation
  - When $0<k<1$ position is intermediate situation
- When $k<0$ or $k>1$ position is external situation

Step №3: calculation

- Linear ocmbination P=(1-k).A+k.B
- Calculation per component

$$P_x = (1\text{-}k).A_x + k.B_x$$
$$P_y = (1\text{-}k).A_y + k.B_y$$
$$P_z = (1\text{-}k).A_z + k.B_z$$

- In the case of angles

$$\alpha_k = (1\text{-}k).\alpha_1 + k.\alpha_2$$
$$\beta_k = (1\text{-}k).\beta_1 + k.\beta_2$$

Example

- The vertices of a triangle are yellow, red and black
- Colour the sides with intermediate colours

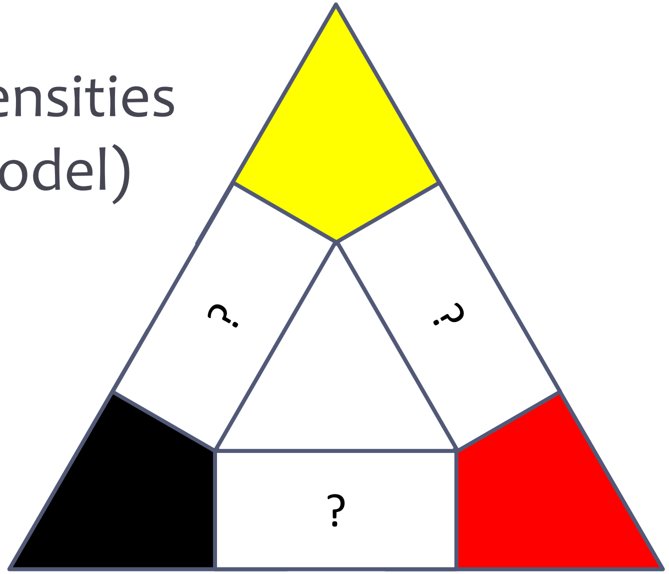
Solution

- Representing colours as triplets of intensities of red, green and blue colours (RGB model)
- The colours at the vertices are

  Yellow (1,1,0)

  Red (1,0,0)

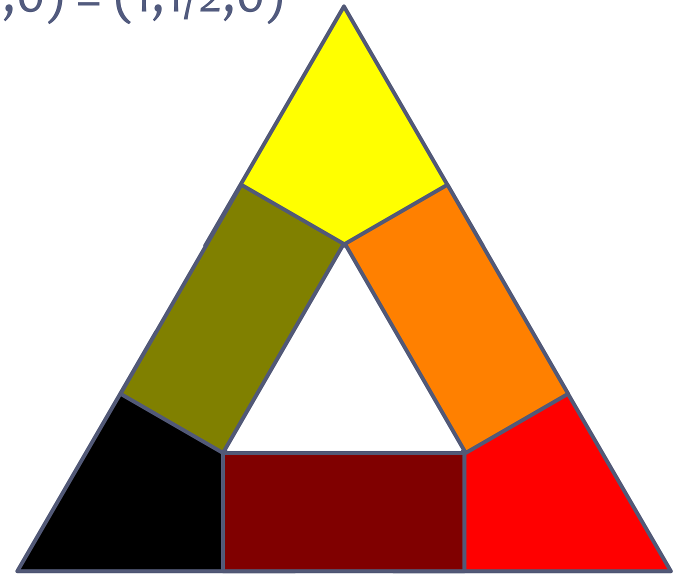
  Black (0,0,0)



Colours of sides

- Linear combination with k=1/2
- Between yellow and red (1,1/2,0)

  Calculated in this way (1-1/2)(1,1,0) + 1/2(1,0,0) = (1,1/2,0)
- Between red and black (1/2,0,0)
- Between black and yellow (1/2,1/2,0)



Range traversal

Problem

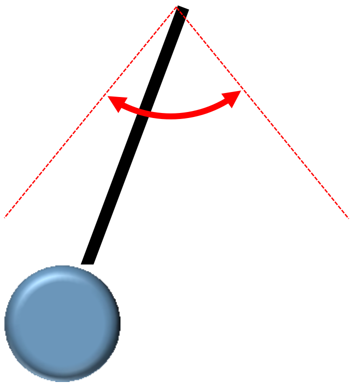
- Numerical parameter is changed within a range
- The user defines how the change occurs

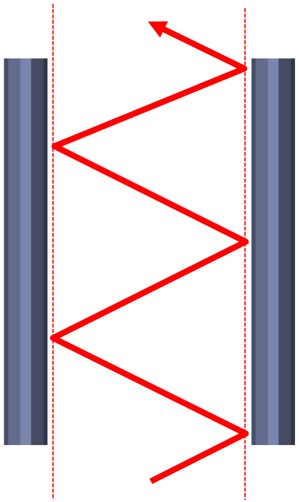
Application

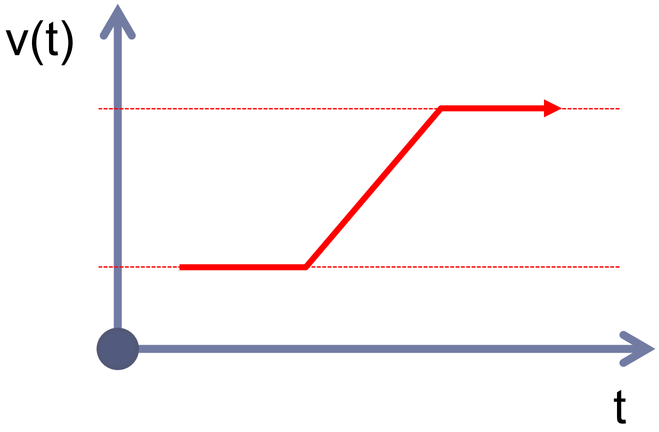
- In animation

Examples

- Pendulum oscilates between two angles
- A ball bounces between two walls
- Motion speed changes from one value to another value







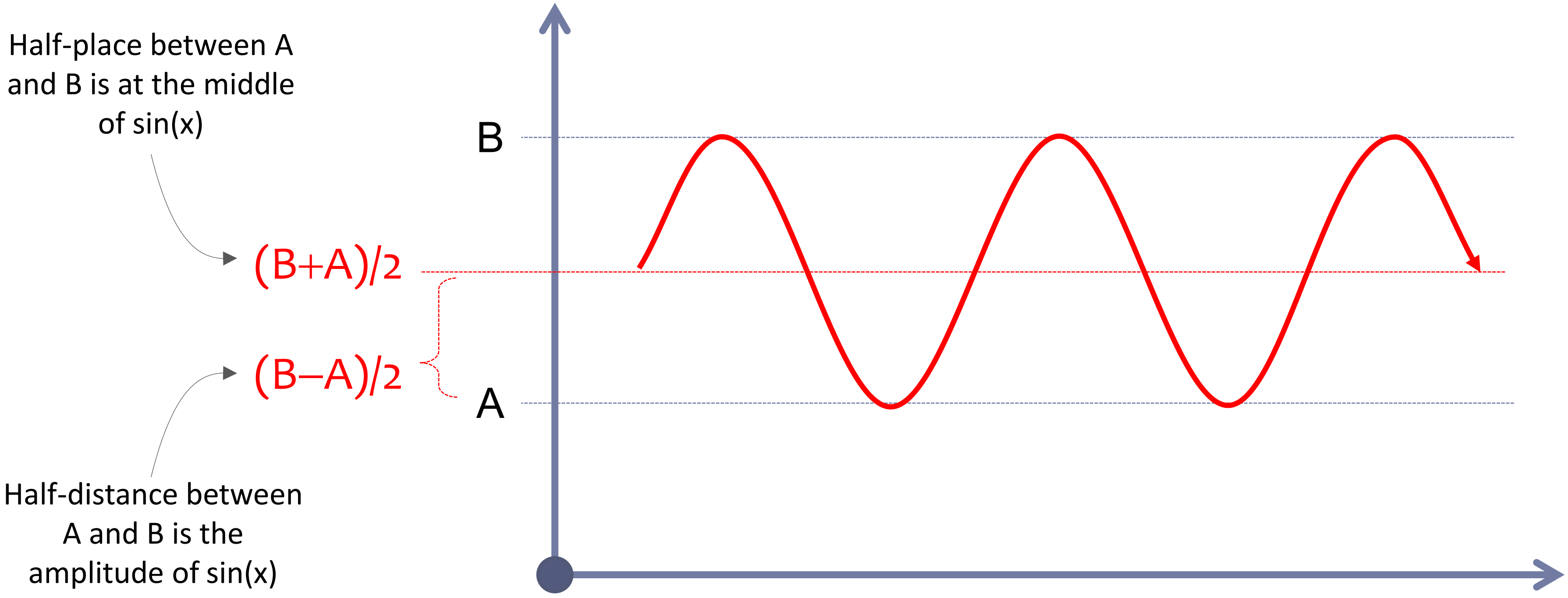
Solution

- Depends on the traversion function
- Let's assume it is $\sin(x)$
- Searching for $f(x)$ that uses $\sin(x)$ but $f(x) \in [A,B]$

Steps

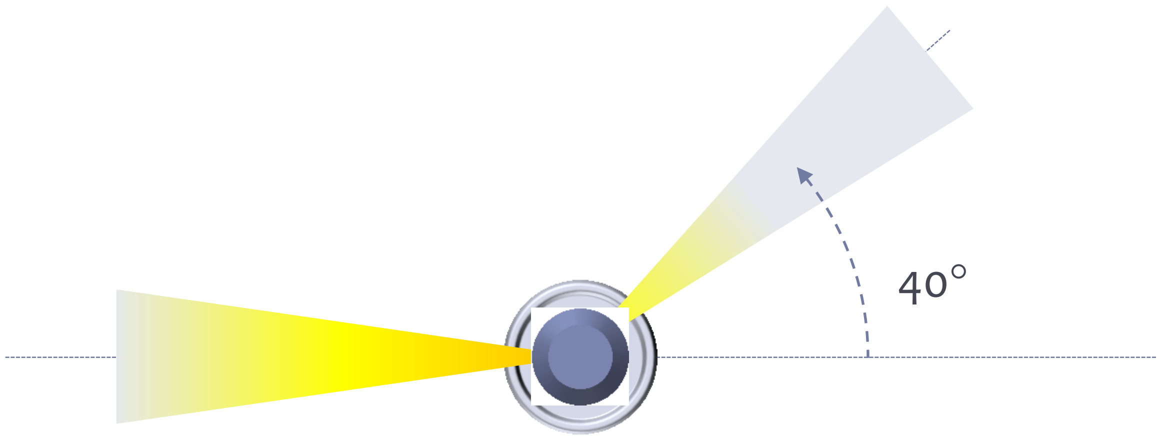
- Initially . . . . $\sin(x) \in [-1,1]$
- Lower boundary 0 . . . . $\sin(x)+1 \in [0,2]$
- Upper boundary 1 . . . . $(\sin(x)+1)/2 \in [0,1]$
- Range B-A . . . . $(B-A).(\sin(x)+1)/2 \in [0,B-A]$
- From A to B . . . . $A+(B-A).(\sin(x)+1)/2 \in [A,B]$
- Transformation . . . . $f(x) = (B+A)/2+(B-A)/2.\sin(x)$

How to remember it?



Example

- Lighthouse's beam rotates left-right
- From 40°
- To 180°
- What is the formula for its motion?

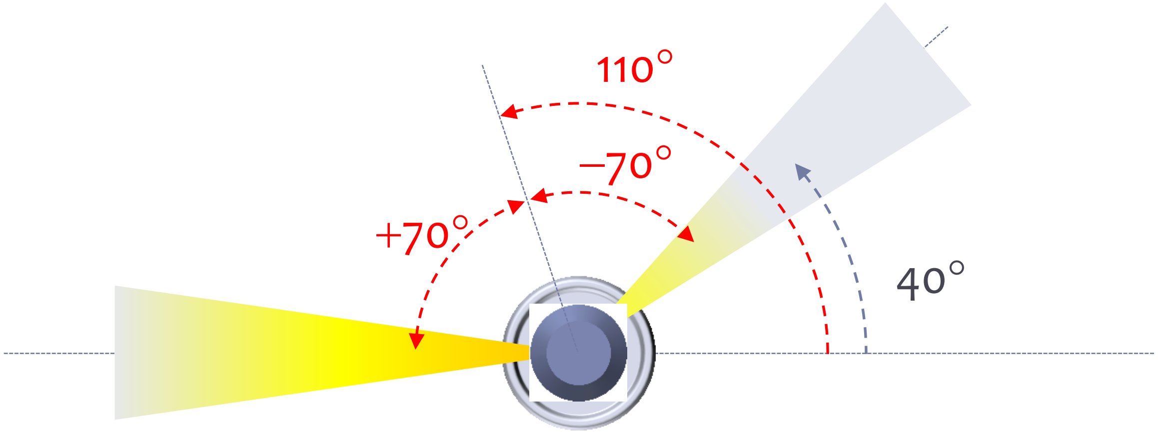


Direct solution

- $\alpha(t) = (180°+40°)/2+(180°-40°)/2.\sin(t) = 110°+70°\sin(t)$

Explanation

- The middle is at 110°, the rotation is ±70° left or right



Summary

Dimensions

Types of dimensions

- Objective (geometrical) – of the object itself
- Spatial – of the space where the object exists
- Visual – of the graphical elements, used to draw the object

Coordinate systems

- Cartesian, coordinates (x,y,z)
- Polar, coordinates (r,α)
- Spherical, coordinates (r,α,β)

Transformation from spherical to Cartesian

- $x = r.\cos(\alpha).\cos(\beta)$
- $y = r.\sin(\alpha).\cos(\beta)$
- $z = r.\sin(\beta)$

From polar to Cartesian

Frequently solved problems

- Translation of object or motion via vector addition
- Distance between two 3D points with the Pythagoras theorem
- Intermediate situation by using linear combination
- Traversal of numerical ranges with sin(x)

ICT in SES

The end

Comments, questions