$f(n) = f_1(n) \pm f_1(n) \pm \dots \pm f_k(n)$
$f_1(n) \succ f_2(n),\ f_1(n) \succ f_3(n),\ \dots\ ,\ f_1(n) \succ f_k(n)$
$\Longrightarrow f(n) \asymp f_1(n)$

$f(n) = c * g(n) \Longrightarrow f(n) \asymp g(n)$
$f(n) \asymp g(n) \Leftrightarrow f(n)^k \asymp g(n)^k$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0 \Leftrightarrow f(n) \prec g(n)$$
$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty \Leftrightarrow f(n) \succ g(n)$$

$$\lim_{n \to \infty} \frac{f(n)}{g(n)} = C > 0 \Rightarrow f(n) \asymp g(n)$$

Ако $g(n)$ е растяща и неограничена и $a > 1,$ то
$f(n) \prec g(n) \Rightarrow a^{f(n)} \prec a^{g(n)}$
Използвано следствие $lg(f(n)) \prec lg(g(n)) \Rightarrow f(n) \prec g(n)$

Ако $f(n)$ и $g(n)$ са растящи и неограничени и $a > 1,$ то
$a^{f(n)} \asymp a^{g(n)} \Rightarrow f(n) \asymp g(n)$

$f(n) \asymp g(n) \Rightarrow lg(f(n)) \asymp lg(g(n))$

$$n! \sim \sqrt{2\pi n}\frac{n^n}{e^n} \Leftrightarrow \lim_{n \to \infty} \frac{\sqrt{2\pi n}\frac{n^n}{e^n}}{n!} = 1$$

Важно следствие $lg(n!) \asymp n\ lg(n)$

$$n! \asymp \sqrt{2\pi n}\frac{n^n}{e^n} \Longrightarrow lg(n!) \asymp lg\left(\sqrt{2\pi n}\frac{n^n}{e^n}\right)$$
$$lg\left(\sqrt{2\pi n}\frac{n^n}{e^n}\right) = lg\left(\sqrt{2\pi}\right) + \frac{1}{2}lg(n) + n\ lg(n) - n\ lg(e) \asymp n\ lg(n)$$
По транзитивността на $\asymp$ следва $lg(n!) \asymp n\ lg(n)$

$a^n \prec n!$

$n \asymp n\, lg(a) = lg\left(a^n\right) \prec lg(n!) \asymp n\, lg(n)$

$n! \prec n^n$

$$\lim_{n \to \infty} \frac{n!}{n^n} = \lim_{n \to \infty} \left(\frac{n}{n} * \frac{n-1}{n} * \ldots * \frac{2}{n} * \frac{1}{n}\right) = 0$$

$$\lim_{n \to \infty} \frac{n^{\epsilon}}{(\log_a n)^k} = \quad // \text{ полагаме } b \leftarrow \frac{\epsilon}{k}$$

$$\lim_{n \to \infty} \frac{\left(n^b\right)^k}{(\log_a n)^k} =$$

$$\lim_{n \to \infty} \left(\frac{n^b}{\log_a n}\right)^k = \quad // k \text{ е положително}$$

$$\lim_{n \to \infty} \frac{n^{\mathrm{b}}}{\log_a \mathrm{n}} = \quad // \text{ прилагаме правилото на l'Hôpital}$$

$$\lim_{n \to \infty} \frac{b n^{b-1}}{\left(\frac{1}{\ln a}\right)\left(\frac{1}{n}\right)} =$$

$$\lim_{n \to \infty} (\ln a) b n^b = \infty$$

Ако $a > 1$ и $k > 0$, то $n^k \prec a^n$

1. $lg\left(n^k\right) = k\, lg(n) \asymp lg(n)$
2. $lg\left(a^n\right) = n\, lg(a) \asymp n$
3. $lg(n) \prec n$ заради горното твърдение

$lg\left(n^k\right) \asymp lg(n) \prec n \asymp lg\left(a^n\right) \Rightarrow n^k \prec a^n$

Да се сравнят функциите $ln(ln(n))$ и $ln(n)$

$$\lim_{n \to \infty} \frac{ln(ln(n))}{ln(n)} = \quad // \text{по правилото на } l'H\hat{o}pital$$

$$\lim_{n\to\infty} \frac{\frac{1}{ln(n)}(ln(n))'}{(ln(n))'} =$$

$$\lim_{n\to\infty} \frac{1}{ln(n)} = 0$$

$$\Rightarrow ln(ln(n)) \prec ln(n)$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$\lg x = \frac{\ln x}{\ln 2} = \frac{1}{\ln 2} \ln x$$

$$f(n) = c{*}g(n) \Longrightarrow f(n) \asymp g(n)$$

$$lg\ x \asymp ln\ x$$

$$lg\ lg\ x \asymp ln\ ln\ x$$

$$lg\ lg\ x = lg\left(\frac{1}{ln\ 2}\ ln\ x\right) = lg\left(\frac{1}{ln\ 2}\right) + lg\ ln(x) = lg\left(\frac{1}{ln\ 2}\right) + \frac{1}{ln\ 2}\ ln\ ln(x)$$

$$lg\ lg\ x = lg\left(\frac{1}{ln\ 2}\right) + \frac{1}{ln\ 2}\ ln\ ln(x)$$

$$lg\ lg\ x \asymp \frac{1}{ln\ 2}\ ln\ ln(x)$$

$$lg\ lg\ x \asymp ln\ ln(x)$$

$$lg\ x * lg\ x \asymp ln\ x * log_c\ x$$

$\lg x \succ lg\ lg\ x \succ lg\ lg\ lg\ x$

// неформално

$f(n) = \sum_{i=0}^{n} 2^i = 1 + 2 + 4 + 8 + \ldots + 2^n$

$1 + f(n) = 1 + 1 + 2 + 4 + 8 + \ldots + 2^n$

$1 + f(n) = \left((((1+1) + 2) + 4) + \ldots 2^n\right)$

$1 + f(n) = \left(((2 + 2) + 4) + \ldots 2^n\right)$

...

$1 + f(n) = 2^n + 2^n$

$f(n) = 2^{n+1} - 1 \asymp 2^{n+1} = 2{*}2^n$

$f(n) \asymp 2^n$

полилогаритмични

$ln\ \ n,\ log_{10}\ n * log_{10}\ n * log_{10}\ n$

полиномиални

$n^2, n^3, n^5 + n^2, \sqrt{n} = n^{1/2},\ n^2 + logn$

експоненциални

$2^n, (3/2)^n,\ 3^{n^5},\ \pi^{n^3+n^2}$

$\frac{1}{n} \prec 1$

$\lim_{n \to \infty} \frac{\frac{1}{n}}{1} = \lim_{n \to \infty} \frac{1}{n} = 0$

0. Функции $\asymp\ n^e,\ e < 0$

1. Констатни функции $f(n)\ = 5,\ f(n) = n^{1/lg(n)} = 2?$

2. Композиране на логаритми $lg^{(k)}x \prec lg^{(l)}x$ при $k > l\ \Longrightarrow ln\ ln\ ln\ ln\ x\ \prec\ ln\ ln\ x \prec ln\ x$

3. Полилогаритмични функции $(lg\ x)^k\ \prec (lg\ x)^l$ когато $k < l$

$\Longrightarrow (lg\ x)^{1/2}\ = \sqrt{lg\ x} \prec lg\ x\ \prec\ lg\ x * lg\ x$

4. Полимониалните функции $\asymp n^k,\ k\ >\ 0$

$n^2,\ n^5\ + n^3 + n\ lgn$

5. Експоненциални функции

$a^n \prec b^n,\ 1 < a <\ b$

6. Факториел

7. 2^{2^n} // хиперекспоненциални