

$$T(n) = 4T(n-2) + n2^n + 4 \cdot 3^n$$

$$x^n = 4x^{n-2}$$

$$x^2 = 4$$

$$x = \pm 2$$

Нехомогенна част

$$\sum_1^k p_i(n) * a_i^n, a_i \neq a_j \text{ при } i \neq j$$

$$T(n) = 4T(n-2) + n2^n + 4 \cdot 3^n + n^2 \cdot 3^n$$

$$T(n) = 4T(n-2) + n2^n + (4 + n^2) \cdot 3^n$$

$$T(n) = c_1 \cdot 3^n + (c_2 + c_3 \cdot n + c_4 \cdot n^2) 2 + c_5 \cdot (-2)^n$$

$$T(n) = T(n-1) + 1$$

$$T(n) = T(n-1) + 1 =$$

$$= T(n-2) + 1 + 1 =$$

$$= T(n-3) + 1 + 1 + 1 =$$

$$= T(n-4) + 1 + 1 + 1 + 1 =$$

.....

$$= T(0) + 1 + 1 + \dots + 1 =$$

$$= T(0) + n \cdot 1 = \theta(n)$$

$$T(n) = T(n-1) + \frac{1}{n}$$

$$\begin{aligned} T(n) &= T(n-1) + \frac{1}{n} = \\ &= T(n-2) + \frac{1}{n-1} + \frac{1}{n} = \\ &= T(n-3) + \frac{1}{n-2} + \frac{1}{n-1} + \frac{1}{n} = \\ &= T(0) + \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{n-1} + \frac{1}{n} = \end{aligned}$$

$$= T(0) + \sum_{i=1}^n \frac{1}{i} = \theta(\lg n)$$

$$T(n) = 2T(n-1) + \frac{1}{n}$$

$$\begin{aligned} T(n) &= 2T(n-1) + \frac{1}{n} = \\ &= 2 \left(2T(n-2) + \frac{1}{n-1} \right) = \\ &= 4T(n-2) + \frac{2}{n-1} + \frac{1}{n} = \\ &= 4 \left(2T(n-3) + \frac{1}{n-2} \right) + \frac{2}{n-1} + \frac{1}{n} = \\ &= 8T(n-3) + \frac{4}{n-2} + \frac{2}{n-1} + \frac{1}{n} = \end{aligned}$$

...

$$= 2^n T(0) + \frac{2^{n-1}}{1} + \dots + \frac{4}{n-2} + \frac{2}{n-1} + \frac{1}{n} =$$

$$= 2^n T(0) + \sum_{i=1}^n \frac{2^{n-i}}{i} =$$

$$\sum_{i=1}^n \frac{2^{n-i}}{i} = 2^n \sum_{i=1}^n \frac{2^{n-i}}{i} \frac{1}{2^n} = 2^n \sum_{i=1}^n \frac{1}{i \cdot 2^i}$$

$$\sum_{i=1}^n \frac{1}{i \cdot 2^i} \leq \sum_{i=1}^n \frac{1}{2^i} \leq \sum_{i=1}^{\infty} \frac{1}{2^i} \leq 1$$

$$2^n \sum_{i=1}^n \frac{1}{i \cdot 2^i} \leq 2^n$$

$$2^n \sum_{i=1}^n \frac{1}{i \cdot 2^i} = O(2^n)$$

$$= 2^n T(0) + \sum_{i=1}^n \frac{2^{n-i}}{i} = 2^n T(0) + 2^n \sum_{i=1}^n \frac{1}{i \cdot 2^i} =$$

$$\theta(2^n) + O(2^n) = \theta(2^n)$$

$$T(n) = \frac{n}{n+1} T(n-1) + 1$$

$$T(n) = \frac{n}{n+1} T(n-1) + 1 =$$

$$= \frac{n}{n+1} \left(\frac{n-1}{n} T(n-2) + 1 \right) + 1 =$$

$$= \frac{n-1}{n+1} T(n-2) + \frac{n}{n+1} + 1 =$$

$$= \frac{n-1}{n+1} \left(\frac{n-2}{n-1} T(n-3) + 1 \right) + \frac{n}{n+1} + 1 =$$

$$= \frac{n-2}{n+1} T(n-3) + \frac{n-1}{n+1} + \frac{n}{n+1} + \frac{n+1}{n+1} =$$

$$\begin{aligned}
& \dots \\
& = \frac{1}{n+1}T(0) + \frac{2}{n+1} + \dots + \frac{n+1}{n+1} = \\
& = \frac{1}{n+1}(T(0) + 2 + 3 + 4 + \dots + n + (n+1)) = \\
& = \frac{1}{n+1}(T(0) - 1 + 1 + 2 + 3 + 4 + \dots + n + (n+1)) = \\
& = \frac{1}{n+1} \left(T(0) - 1 + \frac{(n+1)(n+2)}{2} \right) \asymp \frac{n^2}{n} = n
\end{aligned}$$

$$T(n) = 2T(\sqrt{n}) + 1$$

$$\begin{aligned}
n &= 2^{2^m} \\
m &= \lg \lg n
\end{aligned}$$

$$\begin{aligned}
S(m) = T(n) &= T(2^{2^m}) = 2T(\sqrt{n}) + 1 = 2T\left(n^{\frac{1}{2}}\right) + 1 = \\
&= 2T\left((2^{2^m})^{1/2}\right) + 1 = 2T(2^{2^{m-1}}) + 1 = 2S(m-1) + 1
\end{aligned}$$

$$S(m) = 2S(m-1) + 1$$

$$S(m) = \theta(2^m)$$

$$T(n) = S(m) = \theta(2^m) = \theta(2^{\lg \lg n}) = \theta(\lg n)$$

$$T(n) = T(n-2) + 2 \lg n$$