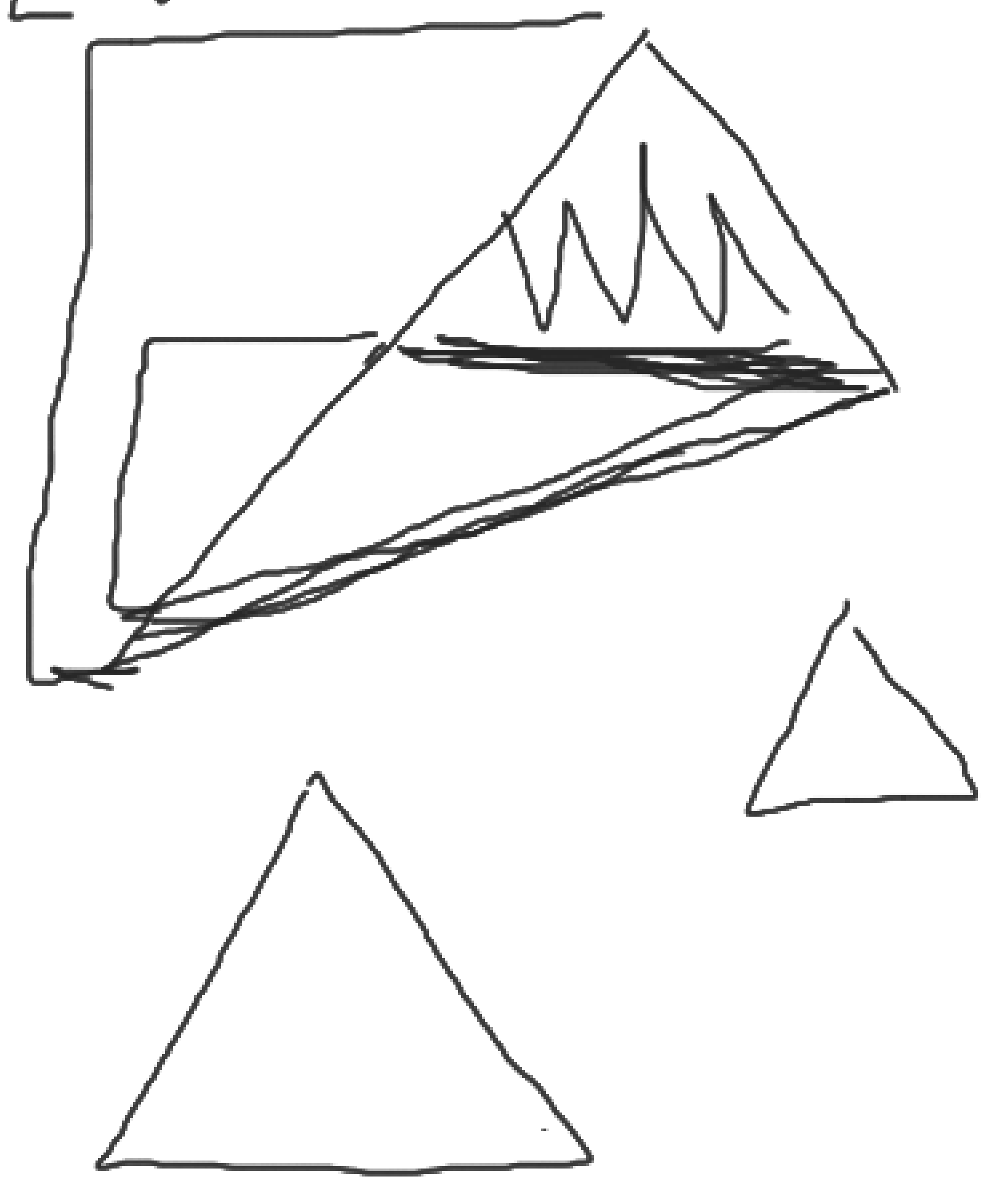
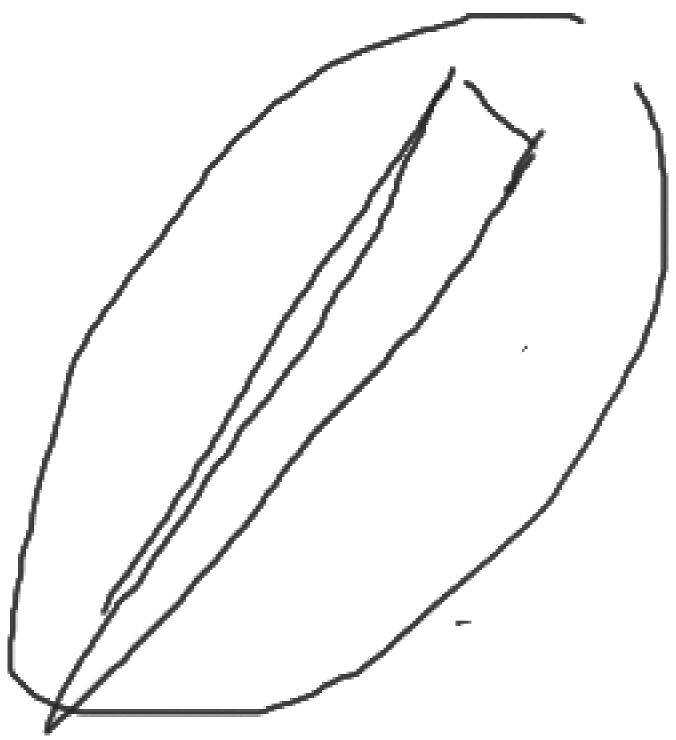
$T(n) = T\left(\frac{n}{2}\right) + \underbrace{T\left(\frac{n}{4}\right)} + \boxed{\lg n}$

$\lfloor \lg n \rfloor$





$\lg n$

$\lg \frac{n}{2}$ $\quad$ $\lg \frac{n}{4}$

$\lg^2 \frac{n}{4}$ $\quad$ $\lg \frac{n}{8}$ $\quad$ $\lg \frac{n}{8}$ $\quad$ $\frac{n}{16}$

$3\ \frac{n}{8}$ $\quad$ $\frac{n}{16}$ $\quad$ $\frac{n}{16}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{16}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{64}$

16, $\quad$ $\frac{n}{16}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{64}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{64}$ $\quad$ $\frac{n}{32}$ $\quad$ $\frac{n}{64}$ $\quad$ $\frac{n}{64}$ $\quad$ $\frac{n}{128}$ $\quad$ $\frac{n}{256}$

$1 \to 1$

$2 \to 1$

$4 \to 2$

$8 \to 3$

$16 \to 5$

$32 \to 8$

$64 \to 13$

$$T(n) = \sum_{i=0}^{\lfloor \lg n \rfloor} f_i \cdot \lg\left(\frac{n}{2^i}\right) = \sum_{i=0}^{\lfloor \lg n \rfloor} f_i \lg n - f_i \underbrace{\lg 2^i} =$$

$$\sum_{i=0}^{\lfloor \lg n \rfloor} f_i \underline{\lg n} - \sum_{i=0}^{\lfloor \lg n \rfloor} f_i \cdot i = \underline{\lg n \sum_{i=0}^{\lfloor \lg n \rfloor} f_i} - \underbrace{\sum_{i=0}^{\lfloor \lg n \rfloor} f_i \cdot i}$$

$$\sum_{j=1}^{k} f_j = \underbrace{f_{k+2}} - 1$$

$$\begin{cases} f_0 = 1 \\ f_1 = 1 \\ f_{n+2} = f_n + f_{n+1} \end{cases}$$

$$x^2 - x - 1 = 0$$

$$x_1 = \frac{1+\sqrt{5}}{2}$$

$$x_2 = \frac{1-\sqrt{5}}{2}$$

$$\asymp \left(\frac{1+\sqrt{5}}{2}\right)^k$$

$$f_n \simeq \left(\frac{1+\sqrt{5}}{2}\right)^n \quad \cancel{2}$$

$$g(n) = \underbrace{\lg n \left(\frac{1+\sqrt{5}}{2}\right)^{\lg n}} \simeq \cancel{\lg n \cdot n}$$

$n = 4^k$ $\quad k-1$ $\quad T(n) = T\left(\frac{n}{a}\right) + T\left(\frac{n}{b}\right) + f(n)$

$T(4^k) = T(\overbrace{2 \cdot 4^{k-1}}) + T(4^{k-1})$ $\quad n = \operatorname{lcm}(a, b)^k = \operatorname{lcm}(2, 4)^k$

k $\quad + \underbrace{\lg(2^{2k})}_{2k}$ $\quad \gcd$ $\quad = 4^k$

$n = 2^k$

$1^k \cdot k$

$T(2^k) = T(2^{k-1}) + T(2^{k-2}) + k$

$S(k) = T(2^k)$ $\quad \boxed{S(k) = S(k-1) + S(k-2)} + k$

$$\left\{ \underline{\frac{1+\sqrt5}{2}},\ \frac{1-\sqrt5}{2},\ 1{,}7\text{M} \right. \to \left(\frac{1+\sqrt5}{2}\right)^k \simeq f(k) = T(2^k),\quad k = \lg n$$

$$\varphi$$

$$T(n) \simeq \left(\frac{1+\sqrt5}{2}\right)^{\lg n}$$

$$T(n) \simeq \lg n \cdot \varphi^{\lg n}$$

$$\boxed{(\exists \alpha \in \mathbb{R}_+)(\exists \beta \in \mathbb{R}_+)(\exists \nu \in \mathbb{N}_+)}(\forall n \in \mathbb{N})\left[n \geq \nu \implies \alpha \cdot \lg n \cdot \varphi^{\lg n} \leq T(n) \leq \beta \cdot \lg n \cdot \varphi^{\lg n}\right]$$

$$(\forall n \in \mathbb{N})\left[(\forall k \in \mathbb{N})[k < n \implies P(k)] \implies P(n)\right]$$

Нека $n \in \mathbb{N}$. Нека за всяко $k \in \mathbb{N}$ ако $k < n$, то $P(k)$. Нека $n \geq r_0$. Тогава $\underline{T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right)} + \lg n$

$\frac{n}{2} < n$ и $\frac{n}{4} < n$. Тогава $\alpha \cdot \lg \frac{n}{2} \cdot \varphi^{\lg \frac{n}{2}} \leq T\left(\frac{n}{2}\right)$ и

$$\alpha \cdot \lg \frac{n}{4} \cdot \varphi^{\lg \frac{n}{4}} \leq T\left(\frac{n}{4}\right)$$

$$T(n) \geq \alpha \cdot (\lg n - 1)\varphi^{\lg n - 1} + \alpha(\lg n - 2)\varphi^{\lg n - 2} + \underline{\lg n}$$

$$\overset{?}{\geq} \alpha \lg n \varphi^{\lg n} = \alpha(\lg n - 1 + 1)\varphi^{\lg n - 1} \cdot \varphi =$$

$$t = \lg n$$

$$T(n) \geq \alpha(t-1)\varphi^{t-1} + \alpha(t-2)\varphi^{t-2} + t \overset{!}{=}$$

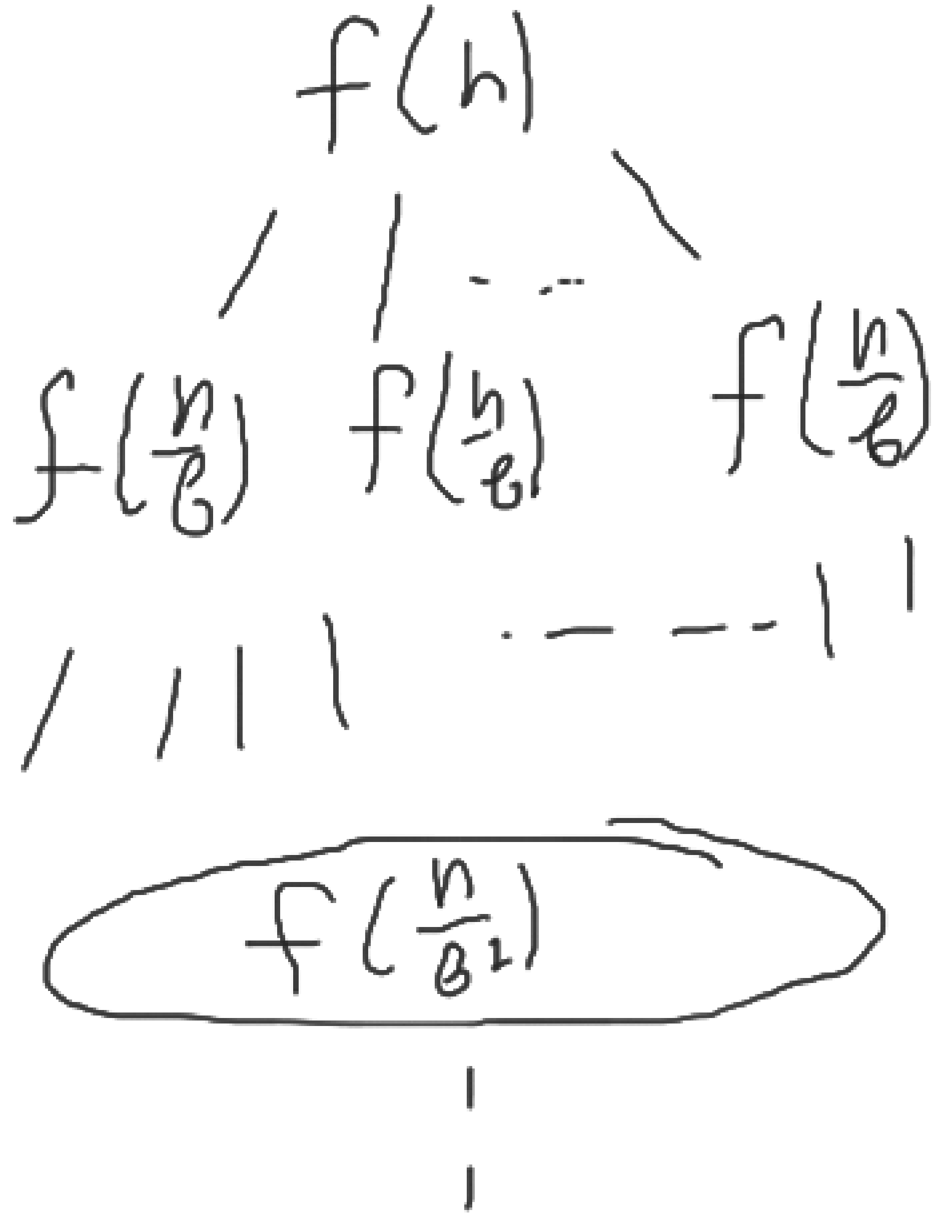
$$\underline{2\alpha t\varphi^{t-2}}(\varphi+1) - \alpha\varphi^{t-1} - 2\alpha\varphi^{t-2} + t \geq \alpha t \varphi^{t}$$

$$\underbrace{2(\varphi+1) - \overset{(0,1)}{\frac{\alpha}{t}}\,\overset{(0,1)}{\overline{(\varphi-2)}} + \overset{(0,1)}{\frac{1}{\alpha\varphi^{t-2}}}}_{(2\varphi,\ 2\varphi+1)} \overset{?}{\geq} \varphi^{2} \quad , \quad t = \lceil \lg n \rceil$$

$$n \geq \sqrt{} \geq 4$$

$$\leq$$

Master theorem



$$T(n) = \underline{aT\left(\frac{n}{b}\right) + f(n)}$$

$0 : f(n)$

$1 : a f\left(\frac{n}{b}\right)$

$2 : a^2 f\left(\frac{n}{b^2}\right)$

$\vdots$

$\ell \quad a^\ell f\left(\frac{n}{b^\ell}\right)$

$$T(n) = \sum_{i=0}^{\lfloor \log_b(n) \rfloor} a^i f\left(\frac{n}{b^i}\right)$$

$c = \log_b a$

$T(n) \simeq n^c = n^{\log_b a}$

1. $\underbrace{f(n) = O(n^{c-\varepsilon})}$, $\varepsilon \in (0,1)$

$c - \varepsilon$

$f(n) \simeq \frac{n^c}{n^\varepsilon}$

$f(n) = \frac{n^c}{n^\varepsilon}$

$$T(n) = \sum_{i=0}^{\lfloor \log_b(n) \rfloor} a^i \left(\frac{n}{b^i}\right)^c = \sum_{i=0}^{l} n^c \left(\frac{a}{b}\right)^i = n^c \underbrace{\sum_{i=0}^{l} \left(\frac{a}{b}\right)^i}$$

$$n^c \left(\frac{\left(\frac{a}{b}\right)^{l+1} - 1}{a/b - 1} \right) = n^c \left(\frac{a^{l+1} - b^{l+1}}{b^{l+1}} \right) \left(\frac{\cancel{b}}{a-b} \right)$$

$$T(n) = n^{l}\left(\frac{a}{b}\right)^{l+1} = n^{\log_b a}\,\frac{a^{l+1}}{b^{l+1}} \qquad a \geq b$$

$$\frac{n^{\log_b a}}{n^{\varepsilon}}\left(\frac{a}{b}\right)^{\log n}$$

maxArr (a[1..n])   VS   maxArr(a[1...n])

$\underline{m \leftarrow 0}$
for i from 1 to n
  if a[i] > m
    m <- a[i]

return m

$\Theta(n)$

$\longrightarrow$

if n = 0
  return 0

if n = 1
  return a[1]

$h \leftarrow \left\lfloor \frac{n}{2} \right\rfloor$

a <- maxArr(a[1,...h])

b <- maxArr(a[h,...n])

return max(a, b)

$$\boxed{\begin{aligned} &T(n) = 2T\left(\tfrac{n}{2}\right) + 1 \\ &a = b = 2 \\ &\log_b a = 1 \\ &1 = n^0 \leq n^{1-\varepsilon} \\ &T(n) \simeq n^{\log_b a} = n \end{aligned}}$$

$T(n) = T\left(\frac{n}{2}\right) + 1$

$a = 1, \quad b = 2 \qquad \log_b a = 0$

$f(n) \simeq 1 \simeq n^0 \qquad T(n) \simeq f(n) \cdot \lg n = \lg n$

$T(b) = T(b/2) + 1$

$$T(n) = 3T\left(\frac{n}{3}\right) + n\lg n$$

$$n^{\log_b a} = n^1 \quad vs \quad n\lg n \qquad f(n) \geq n^1$$

$$\neq$$

$$? \quad f(n) = O(n^{1-\varepsilon}) \qquad n\lg n \overset{?}{\leq} n^{1-\varepsilon} \quad \times$$

$$? \quad f(n) = \Omega(n^{1+\varepsilon}) \qquad n^{1+\varepsilon} \overset{?}{\leq} n\lg n \quad \text{sum}$$

$$\boxed{n^{\varepsilon}} \overset{?}{\leq} \lg n \quad \times$$

$$T(n) = 3T\left(\frac{n}{3}\right) + n\lg n = 3\left(3T\left(\frac{n}{9}\right) + \frac{n}{3}\lg\frac{n}{3}\right) + n\lg n$$

$$= 9T\left(\frac{n}{9}\right) + n\lg\frac{n}{3} + n\lg n =$$

$$9\left(3T\left(\frac{n}{27}\right) + \frac{n}{9}\lg\frac{n}{9}\right) + n\lg\frac{n}{3} + n\lg n =$$

$$27T\left(\frac{n}{27}\right) + n\lg\frac{n}{9} + n\lg\frac{n}{3} + n\lg n =$$

$$3^{3}T\left(\frac{n}{3^3}\right) + n\left(3\lg n - \overline{\lg 9} - \overline{\lg 3} - \overline{\lg 1}\right)$$

$$T(n) = \underbrace{3^{\log_3 n}}_{n} \cdot \underbrace{T(0)}_{C} + n\left(\log_3 n \cdot \lg n - \sum_{i=0}^{\log_3 n - 1} \lg 3^i\right)$$

$$\sum_{i=0}^{\ell-1} \lg 3^i = \sum_{i=0}^{\ell-1} i \lg 3 = \lg 3 \cdot \frac{(\ell+1)\ell}{2}$$

$$T(n) \simeq n + \underbrace{n \lg n \cdot \log_3 n} - \log_3^2 n$$

$$\simeq n \lg^2 n$$

$$\boxed{\log_3 n}$$

$n \lg n$

$+$

$3 \frac{n}{3} \lg \frac{n}{3}$

$+$

$9 \frac{n}{9} \lg \frac{n}{9}$

$+$

$27 \frac{n}{27} \lg \frac{n}{27}$

$+$

$1 \cdot 3^{\log_3 n}$

$l: n \lg \frac{n}{3^l}$

$n \lg n$

$n(\underline{\lg n - l \lg 3})$

$\asymp \lg n (n \lg n)$

$$T(n) = 8T\left(\frac{n}{2}\right) + \frac{n^3}{\lg n}$$

1. ×
2. ×
3. ×

$$a = 8,\quad b = 2,\quad f(n) = \frac{n^3}{\lg n}$$

$$c = \log_b a = 3$$

$$n^c \neq f(n)$$

$f(n) = O(n^{3-\varepsilon})$	$\frac{n^3}{\lg n} \leq \frac{n^3}{n^\varepsilon}$
$f(n) = \Omega(n^{3+\varepsilon})$	$n^3 \cdot n^\varepsilon \leq \frac{n^3}{\lg n}$

$$T(n) = 2T\left(\frac{n}{2}\right) + n^2$$

n^1 vs n^2

$n^2 \stackrel{?}{=} \Omega\left(n^{1+\varepsilon}\right)$

$af\left(\frac{n}{b}\right) \le d\,f(n)$

$? \; d \in (0,1)$

$2\left(\frac{n}{2}\right)^2 \le dn^2$

$\frac{n^2}{2} \le dn^2$

$d = \frac{1}{2}$ ✓

✓ $n^{1+\varepsilon} \le n^2$? $\lim_{n \to \infty} \frac{n^2}{n^{1+\varepsilon}} = \infty$ $\varepsilon \in (0,1)$

$\mathrm{D}\!+$ 3ca M.T $T(n) \asymp n^2$