$$g(x,y,z) \simeq \begin{cases} x+y, & z=0 \,\&\, (x=0 \vee y=0) \\ 2+g(x-1,y-1,0), & z=0 \,\&\, x>0 \,\&\, y=0 \\ x+y+g(x,y,z-1), & z>0 \end{cases}$$

$$(\forall x\in\mathbb{N})(\forall y\in\mathbb{N})(\forall z\in\mathbb{N})[g(x,y,z)=(z+1)(x+y)]$$

$$\langle a,b,c\rangle \sqsubset \langle k,l,m\rangle \iff \begin{cases} c<m \;\vee \\ c=m \,\&\, b<l \;\vee \\ c=m \,\&\, b=l \,\&\, a<k \end{cases}$$

$$\sqsubset$$

$$\langle \mathbb{N}^3, <_{a\text{-}lex}\rangle$$

$$\ldots \sqsubset a_2 \sqsubset a_1 \sqsubset a_0$$

$(\forall \langle x,y,z\rangle \in \mathbb{N}^3)\Big[(\forall \langle a,b,c\rangle \in \mathbb{N}^3)\big[\langle a,b,c\rangle \sqsubset \langle x,y,z\rangle \Longrightarrow g(a,b,c) = (c+1)(a+b)\big]$

$\langle \mathbb{N}^3, \sqsubset\rangle$ $\qquad \Longrightarrow g(x,y,z) = (z+1)(x+y)\Big]$ Д-во

Нека $x,y,z \in \mathbb{N}$. Нека за всеки $a,b,c \in \mathbb{N}$ ако $\langle a,b,c\rangle \sqsubset \langle x,y,z\rangle$, то $g(a,b,c) = (c+1)(a+b)$.

Вззм. са 3 сл. според деф. g.

Сл. 1 $z=0$ и $x=0$ или $y=0$. Тогава $g(x,y,z) \overset{\text{деф}}{=} x+y$

$= (x+y)\cdot 1 = (0+1)(x+y) = (z+1)(x+y)$. ✓

Сл. 2. $z = 0$ и $x > 0$ и $y = 0$. Тогава $g(x,y,z) \overset{\text{деф}}{\simeq} 2 + g(x-1, y-1, 0)$

$= 2 + g(x-1, y-1, z) \overset{\text{И.Х.}}{=} 2 + (z+1)(x-1+y-1) = (z+1)(x - \cancel{1} + y - \cancel{1} + \cancel{2})$

$\langle x-1, y-1, z\rangle \sqsubset \langle x, y, z\rangle$ $\qquad = (z+1)(x+y).$ ✓

Сл. 3 $z > 0$. Тогава $g(x,y,z) \overset{\text{деф}}{\simeq} x + y + g(x, y, z-1) =$

$x + y + (z - \cancel{1} + \cancel{1})(x+y) = (z+1)(x+y).$ ✓ $\qquad x \mapsto y \mapsto z \mapsto \ldots$

Така $g(x,y,z) = (z+1)(x+y).$

След $g = \langle x, y, z\rangle \mapsto (z+1)(x+y) = \lambda x \lambda y \lambda z[(z+1)(x+y)]$

$$h(x) \simeq \begin{cases} x+1, & x<2 \\ h(x+1), & x \geq 2 \end{cases}$$

$h(0)=1,\ h(1)=2,$

$x=2+n,\quad h(x) \simeq h(x+1) = h(2+n+1) \simeq h(n+4) \simeq$

$h(n+5) \simeq \ldots$

$T(n) = 1 + T(\lfloor \frac{n}{2} \rfloor) + 4 = T(\lfloor \frac{n}{2} \rfloor) + 5$

1

$\cancel{T}(0) = 2 \quad 1 \leftarrow$

$$T(n) = \begin{cases} 2, & n = 0 \\ T(\lfloor n/2 \rfloor) + 5, & n > 0 \end{cases}$$

$T(0) = C \geq 1$

$T(1) = A \geq ?$

Cn. 1 $\quad n = 2^k$. $\quad T(n) = T(4 \cdot 2^l) = T(2 \cdot 2^l) + 5$

$= T(2^l) + 5 + 5 = T(2^l) + 2 \cdot 5$

$k \geq 2$

$k = 2 + l \qquad = T(2^{l-1}) + 3 \cdot 5 = T(2^{k-3}) + 3 \cdot 5$

$$T(2^m) \overset{m}{=} T(0) + m \cdot 5 = 2 + m \cdot 5 = 2 + \lg(2^m) \cdot 5$$

$\underline{n = 2^k \cdot 3}$ $\quad T(n) = T\left\lfloor \frac{n}{2} \right\rfloor + 5 = T\left\lfloor \frac{2^k \cdot 3}{2} \right\rfloor + 5 =$

$$T(2^{k-1}) + 5 = (2 + (k-1) \cdot 5) + 5 = 2 + k \cdot 5$$

$$= 2 + \lfloor \lg n \rfloor \cdot 5$$

$q > 3$

$$n = 2^k \cdot q, \quad q \equiv 1 \rightarrow T(n) = 2 + \lfloor \lg n \rfloor \cdot 5$$

$$\rightarrow n = 2^{p_0} + 2^{p_1} + 2^{p_2} + \ldots (2^{p_n})$$

$$\lg 2^k \cdot 3 = \lg 2^k + \lg 3 = k + 1 + \varepsilon$$

$$\lfloor \lg n \rfloor = k+1$$

$$T(n) = \lfloor \lg n \rfloor 5 + 2 \asymp \lfloor \lg n \rfloor \qquad T: \mathbb{N}_+ \to \mathbb{N}_+$$

$$T: \mathbb{R}_+ \to \mathbb{R}_+, \quad T(n) = \underbrace{5}_{}\lg n + \underbrace{2}_{} \asymp \underline{\lg n}$$

$T(n) = T(n/2) + 5$

$T(n) = T(n/4) + 2 \cdot 5 = T\left(\frac{n}{8}\right) + 3 \cdot 5 = \ldots T\left(\frac{n}{2^{\lg n}}\right) + \lg n \cdot 5$

$\ldots$

$(0, 1)$

$= C + 5 \lg n \simeq \lg n$

$T : \mathbb{N} \to \mathbb{R}_+$

$$T(n) = \underline{T(n-1) + T(n-2)} + \underbrace{\left(\frac{n^3+3n^2}{n}\right)^n + 2^n n^2 + 5^n}_{\text{нехомог. част}}$$

$T(0) = \underline{A}$, $T(1) = B \geq 1$

1. Разг. хом. част
2. Характ. у-ние $\to$ P_d
3. Нам. корените (реш.)

$\to$ $x_1, x_2, \dots, x_k$ \
$r_1, r_2, \dots, r_k$

$r_1 + r_2 + \dots + r_k = d$

4. Доб. / присъед.. корени от нехом. част
5. Реш. система

$$\underbrace{Q_1(n)\cdot C_1^n + Q_2(n)\cdot C_2^n + \dots + Q_l(n)\cdot C_l^n}_{\text{квазиполином}}.$$

$\deg Q_1, C_1$ $\deg Q_2, C_2, \dots,$ $\deg Q_l, C_l$

$y_1, y_2, \dots, y_v$

$s_1, s_2, \dots, s_v$

$\rightarrow R_1(n)\cdot y_1^n + R_2(n)\cdot y_2^n + \dots$

s_1 s_2

$$(n^2+n+1)5^n + \left(\frac{3}{2}n^3+7n\right)\cdot 9^n \asymp n^2 5^n + n^3 9^n \asymp n^3 9^n$$

$$n^2 5^n \qquad \frac{3}{2}n^3 9^n$$

$$W(n)\cdot 5^n + H(n)\cdot 9^n \asymp n^2 5^n + n^3\cdot 9^n \asymp \boxed{n^3 9^n}$$

$$2 \qquad 3$$

$$\lim_{n \to \infty} \frac{n^3 \cdot 9^n}{n^{10} \cdot 5^n} = \lim \frac{\left(\frac{9}{5}\right)^n}{n^7} = \infty$$

$A > B$

$$n^k \cdot \underbrace{A}^n \quad \text{vs} \quad n^l \cdot \underbrace{B}^n \implies n^k A^n$$

$A = B$, $k > l$

$$\implies n^k \cdot A^n$$

$$T(n) = T(n-1) + T(n-2) + \left[(n^2+3n)1^n + n^2 \cdot 2^n + 1 \cdot 5^n\right]$$

$$x^d = x^{d-1} + x^{d-2} \quad | \, x^{d-2}$$

$$x^2 = x + 1$$

$$x^2 - x - 1 = 0$$

$$D = 1 + 4 = 5$$

$$x_1 = \frac{1+\sqrt{5}}{2}, \quad x_2 = \frac{1-\sqrt{5}}{2} < 0$$

$\nearrow 1$ $\qquad 1$

$2, 1 \mid 2, 2 \mid 0, 5$

$\downarrow \qquad \downarrow \qquad \downarrow$

$3 \times 1 \qquad 3 \times 2 \qquad 1 \times 5 \quad | \; 1 \times \varphi_1, \; 1 \times \varphi_2$

$$T(n) \asymp n^2 \cdot 1^n + n^2 2^n + 1 \cdot 5^n + \varphi_1^n + \varphi_2^n$$

$$\asymp 5^n$$

$$T(n) = 2\sum_{i=0}^{n-1} T(i) + \boxed{n^2}$$

пойти истрия

$f(n) \to f(n-1), f(n-2), \ldots, f(0)$ $\Big\}$ $0, \ldots, n-1$

for
for $\Big\}$ n^2

$T(n) - T(n-1) =$

$$2\sum_{i=0}^{n-1} T(i) + n^2 - \left(2\sum_{i=0}^{n-2} T(i) + (n-1)^2\right) =$$

$$2T(n-1) + \cancel{n^2} - \cancel{n^2} + 2n - 1$$

$$\longrightarrow T(n) = 3T(n-1) + (2n-1)1^n$$

1. $x = 3$

2. 2×1

3. 1×3 , $1^{2 \times 1}$

4. $T(n) \asymp 1 \cdot 3^n + n \cdot 1^n \asymp 3^n$

$T(n) = 3T(n-1)$

$x^n = 3x^{n-1} \mid x^{n-1}$

$x = 3 \rightarrow 1 \times 3$

$(2n-1) 1^n$

$d = 1$

2×1

$+1$

-1

$T = \Theta(3^n)$

$\in$