$$T(n) = \underbrace{\sum_{i=0}^{n-2} T(i)}_{} + n^2 \quad \Big| + T(n-1)$$

$$\to \quad T(n) = T(n-1) + T(n-2) + 2n-1$$

$$T(n) + \underbrace{T(n-1)}_{} = \sum_{i=0}^{n-1} T(i) + n^2$$

$n-2, n-1$

$$T(n) + \left(\sum_{i=0}^{n-3} T(i) + (n-1)^2\right) = \sum_{i=0}^{\cancel{n-1}} T(i) + n^2$$

$$T(n) = T(n-1) + T(n-2) + n^2 - (n-1)^2 \to 2n-1$$

$$\underbrace{a_n = a_{n-1} + a_{n-2}} + \underbrace{2n-1}$$

1. $a_n - a_{n-1} - a_{n-2} = 0$

2. $x^n - x^{n-1} - x^{n-2} = 0 \quad \| (x^{n-2})^{-1}$

$x^2 - x - 1 = 0$

$D = 1 + 4 = 5$

$\underbrace{x_1} = \frac{1+\sqrt{5}}{2}, \underbrace{x_2} = \frac{1-\sqrt{5}}{2}$

3. $2n-1 = \overbrace{(2n-1)}^{2} \cdot \underbrace{1^n}$

4. $\{\underbrace{x_1}, x_2, \underline{1}, \underline{1}\}_M$

5. $\underbrace{Ax_1^n} + Bx_2^n + (Cn+D)1^n$

6. $Ax_1^n + Bx_2^n + Cn \cdot 1^n$

7. $\underbrace{A}\left(\frac{1+\sqrt{5}}{2}\right)^n$

$T(n) \simeq \left(\frac{1+\sqrt{5}}{2}\right)^n$

Say. $S(n) = S(\frac{n}{2}) + S(\frac{n}{4}) + S(\frac{n}{8}) + \dots + S(1) + n$

$n = 2^m$, $m = \lfloor \lg n \rfloor$

$S(2^m) = S(2^{m-1}) + S(2^{m-2}) + \dots + S(2^0) + 2^m$

$T(m) = S(2^m)$

$$T(m) = \sum_{i=0}^{m-1} T(i) + 2^m \quad \Big| - T(m-1)$$

$T(m) - T(m-1) = T(m-1) + 2^m - 2^{m-1}$

$$T(n) = 2T(n-1) + (2-1)2^{n-1} = \underbrace{2T(n-1) + \frac{1}{2}\cdot 2^n}$$

---

1. $x = 2$

2. $\frac{1}{2} \cdot 2^n \rightarrow 1 \times 2$

3. $\{2, 2\}_M$

4. $(An+B)2^n = An2^n + B2^n$

5. $An2^n$.

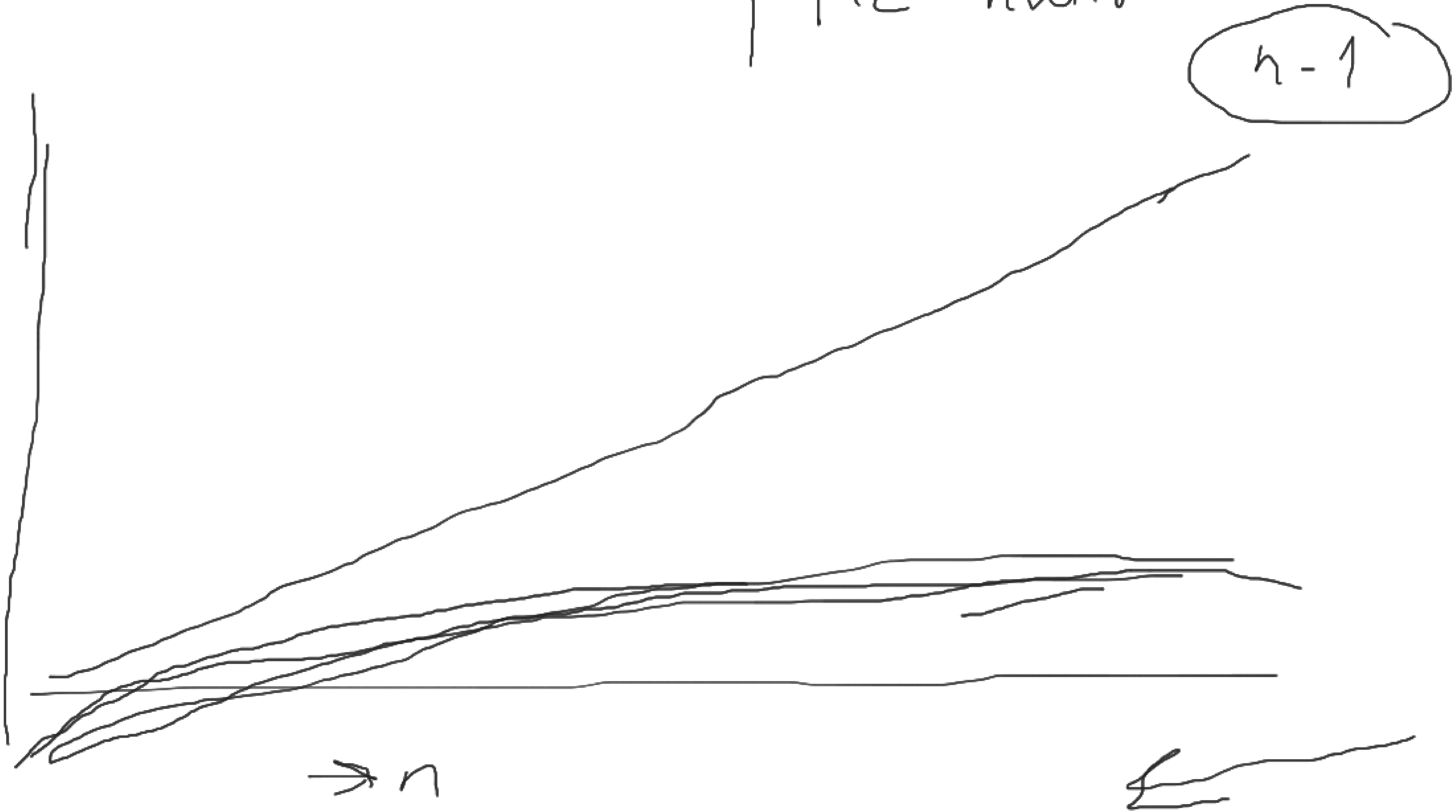
6. $T(n) \simeq n \cdot 2^n$

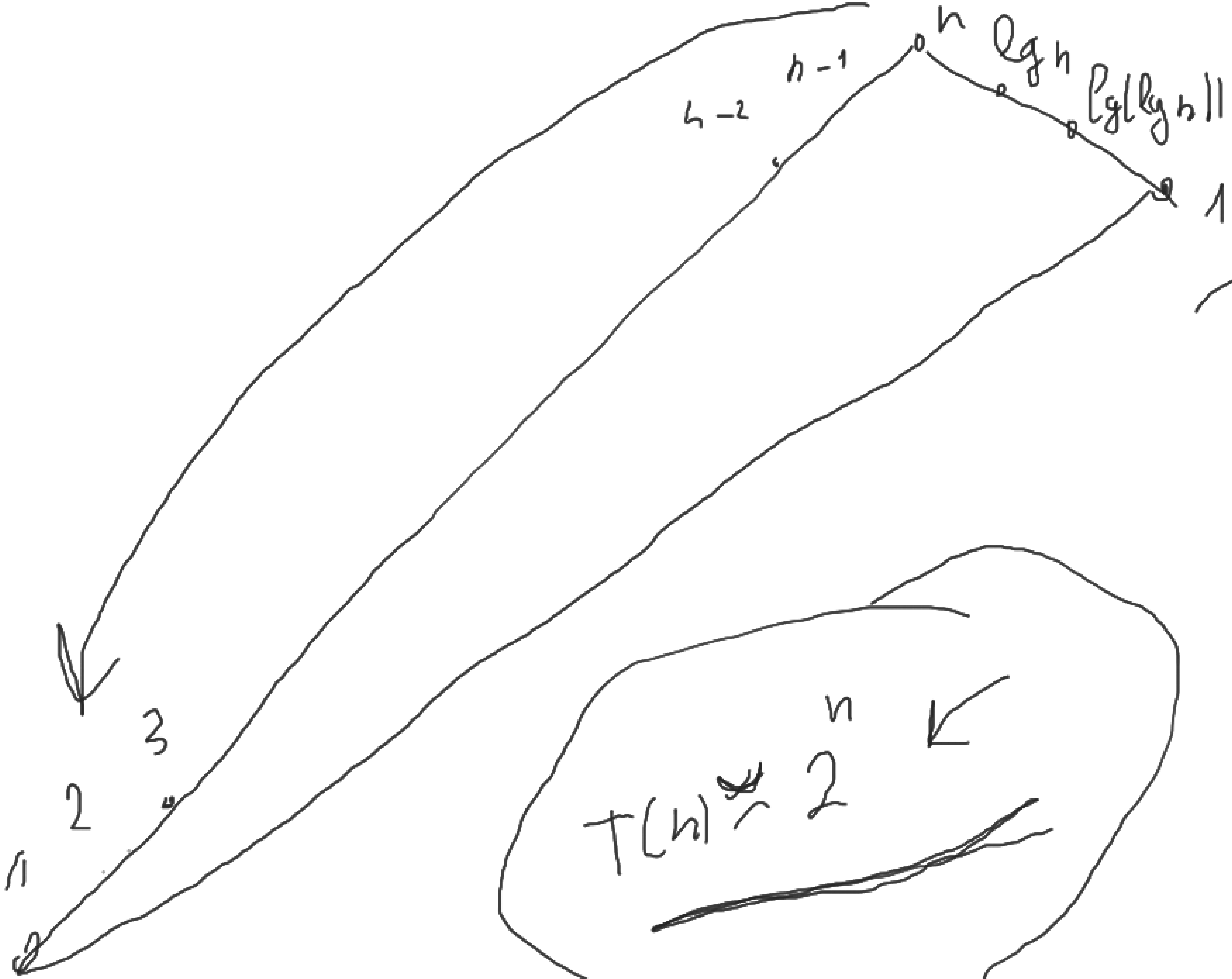
$$\boxed{T(m) = S(2^m), \quad m = \lg n \quad 2^m = n}$$

$\hookrightarrow S(n) \simeq \lg n \cdot n$

Зад 3. $T(n) = 2T(n-1) + T(\lg n) + n$

↑ Не лин.





$T(n) \simeq R(n)$

$R(n) = 2R(n-1) + n$

$(n+0)1^n$

$x = 2$

$\{2, 1, 1\}$

$A2^n + Bn + C$

$\simeq 2^n$

$R(n) \simeq 2^n$

$T(n) \simeq 2^n$

Пр. $T(n) = 2T(n-1) + T(\lg n) + n \asymp 2^n$

$(\forall n \in \mathbb{N}) \underbrace{(\exists \alpha \in \mathbb{R}^+)(\exists \beta \in \mathbb{R}^+)[\alpha 2^n \leq T(n) \leq \beta 2^n]}_{P(n)}$

$T(n)$

$T(n-1)$

База: $T(0) = C$ | ? $\alpha 2^0 \leq C \leq \beta \cdot 2^0$

$\alpha \leq C \leq \beta$

$(\exists \alpha \in \mathbb{R}^+)(\exists \beta \in \mathbb{R}^+)(\exists \nu \in \mathbb{N}^+)(\forall k \in \mathbb{N})[k \geq \nu \Rightarrow \alpha 2^k \leq T(k) \leq \beta 2^k]$

$T(n) = 2T(n-1) + \Theta(\lg n) + n \asymp 2^n$

Приемаме, че са ни дадени положителни $\alpha, \beta \in \mathbb{R}^+$ и $\nu \in \mathbb{N}^+$. По-късно ще ги определим. По индукция ще док.

$$(\forall n \in \mathbb{N}) \underbrace{\left[n \geq \nu \implies \alpha 2^n \leq T(n) \leq \beta 2^n \right]}_{P(n)}$$

$$(\forall l \in \mathbb{N})\left[(\forall u \in \mathbb{N})\left[\underline{u < l} \implies P(u)\right] \implies (\forall l \in \mathbb{N})\, P(l)\right]$$

Нека $n \in \mathbb{N}$. Нека е в сила. За всяко $s \in \mathbb{N}$,

ако $s < n$, то ако $s \geq \nu$ да е в сила

$$\alpha 2^s \leq T(s) \leq \beta 2^s.$$

Искаме да видим, че ако $n \geq \nu$, то

$$\alpha 2^n \leq T(n) \leq \beta 2^n.$$

Нека $n \geq \nu$. Тогава със сл. е в сила. е

$$T(n) = 2T(\underbrace{n-1}_{<n}) + T(\underbrace{\lg n}_{<n}) + n,$$ защото $\nu \geq 1$.

Тогава $\alpha 2^{n-1} \leq T(n-1) \leq \beta 2^{n-1}$ и

$$\alpha 2^{\lg n} \leq T(\lg n) \leq \beta \underbrace{2^{\lg n}} \quad \text{т.О}$$

$$\alpha n \leq T(\lg n) \leq \beta n.$$ Значи

$$T(n) = 2T(n-1) + T(\lg n) + n \geq 2(\alpha 2^{n-1}) + \alpha n + n$$

$$= \alpha 2^n + \overset{\geq n}{(\alpha+1)n} \geq \alpha 2^n.$$ Тоест $\alpha 2^n \leq T(n)$

Остава $T(n) \le B2^n$

$$T(n) = 2T(n-1) + T(\lg n) + n \le 2B2^{n-1} + Bn + n =$$

$$B2^n + (B+1)n \overset{?}{\le} B2^n$$

$$\cancel{B2^n} + (B+1)n \le \cancel{B2^n}$$

$$(B+1)\cancel{n} \le 0$$

$$B+1 \le 0$$

$$B \le -1$$

$$B2^x + (B+1)x \le 2^x$$

$$\cancel{x > 5}$$

$$B2^x + (B+1)x \le C2^x$$

$$(B+1)x \le (C-B)2^x \,\Big|\lg$$

$$\lg[(\beta+1)x] \le \lg[(c-\beta)2^x]$$

$$\lg(\beta+1) + \lg x \le \lg(c-\beta) + x$$

$$\lg(\beta+1) \le \lg(c-\beta) \quad \Big|\, 2^x$$

$$\beta+1 \le c-\beta$$

$$2\beta \le c-1$$

$$\beta \le \frac{c-1}{2}$$

Изпълнява

$T(n) \le \beta 2^n$, ако

$$\beta \le \frac{c-1}{2}$$

$$\boxed{\text{Ако } \alpha=1,\ \gamma=\delta,\ \beta=\frac{c-1}{2}}$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + \lg n$$