ГРАНИЦИ НА РЕДИЦИ.

РЕШЕНИ ЗАДАЧИ.

зад.1. Пресметнете: $\lim\limits_{n\to\infty}\frac{n-1}{n+1}=?$

$$\lim_{n\to\infty}\frac{n-1}{n+1}=\lim_{n\to\infty}\frac{n\left(1-\frac{1}{n}\right)}{n\left(1+\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{1-\frac{1}{n}}{1+\frac{1}{n}}=\frac{\lim\limits_{n\to\infty}\left(1-\frac{1}{n}\right)}{\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)}=$$

$$=\frac{\lim\limits_{n\to\infty}1-\lim\limits_{n\to\infty}\frac{1}{n}}{\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{1}{n}}=\frac{1-0}{1+0}=\frac{1}{1}=1$$

зад.2. Пресметнете: $\lim_{n\to\infty}\frac{2n+1}{3n+4}=?$

$$\lim_{n\to\infty}\frac{2n+1}{3n+4}=\lim_{n\to\infty}\frac{n\left(2+\frac{1}{n}\right)}{n\left(3+\frac{4}{n}\right)}=\lim_{n\to\infty}\frac{2+\frac{1}{n}}{3+\frac{4}{n}}=\frac{\lim_{n\to\infty}\left(2+\frac{1}{n}\right)}{\lim_{n\to\infty}\left(3+\frac{4}{n}\right)}=$$

$$=\frac{\lim_{n\to\infty}2+\lim_{n\to\infty}\frac{1}{n}}{\lim_{n\to\infty}3+\lim_{n\to\infty}\frac{4}{n}}=\frac{2+0}{3+0}=\frac{2}{3}$$

зад.3. Пресметнете: $\lim\limits_{n\to\infty}\dfrac{2n^2-1}{n^2+4}=?$

$$\lim_{n\to\infty}\frac{2n^2-1}{n^2+4}=\lim_{n\to\infty}\frac{n^2\left(2-\frac{1}{n^2}\right)}{n^2\left(1+\frac{4}{n^2}\right)}=\lim_{n\to\infty}\frac{2-\frac{1}{n^2}}{1+\frac{4}{n^2}}=\frac{\lim\limits_{n\to\infty}\left(2-\frac{1}{n^2}\right)}{\lim\limits_{n\to\infty}\left(1+\frac{4}{n^2}\right)}=$$

$$=\frac{\lim\limits_{n\to\infty}2-\lim\limits_{n\to\infty}\frac{1}{n^2}}{\lim\limits_{n\to\infty}1+\lim\limits_{n\to\infty}\frac{4}{n^2}}=\frac{2-0}{1+0}=\frac{2}{1}=2$$

зад.4. Пресметнете: $\lim\limits_{n\to\infty}\dfrac{2n+3}{n^2+2n-4}=?$

$$\lim_{n\to\infty}\frac{2n+3}{n^2+2n-4}=\lim_{n\to\infty}\frac{n\left(2+\frac{3}{n}\right)}{n^2\left(1+\frac{2}{n}-\frac{4}{n^2}\right)}=\lim_{n\to\infty}\frac{2+\frac{3}{n}}{n\left(1+\frac{2}{n}-\frac{4}{n^2}\right)}=$$

$$=\lim_{n\to\infty}\left(\frac{1}{n}\cdot\frac{2+\frac{3}{n}}{1+\frac{2}{n}-\frac{4}{n^2}}\right)=\lim_{n\to\infty}\frac{1}{n}\cdot\lim_{n\to\infty}\left(\frac{2+\frac{3}{n}}{1+\frac{2}{n}-\frac{4}{n^2}}\right)=0\cdot\lim_{n\to\infty}\left(\frac{2+\frac{3}{n}}{1+\frac{2}{n}-\frac{4}{n^2}}\right)=0$$

зад.5. Пресметнете: $\lim\limits_{n\to\infty}\dfrac{2n^3+3}{n^4+2n^2-4}=?$

$$\lim_{n\to\infty}\frac{2n^3+3}{n^4+2n^2-4}=\lim_{n\to\infty}\frac{n^3\left(2+\frac{3}{n^3}\right)}{n^4\left(1+\frac{2}{n^2}-\frac{4}{n^4}\right)}=\lim_{n\to\infty}\frac{2+\frac{3}{n^3}}{n\left(1+\frac{2}{n^2}-\frac{4}{n^4}\right)}=$$

$$=\lim_{n\to\infty}\left(\frac{1}{n}\cdot\frac{2+\frac{3}{n^3}}{1+\frac{2}{n^2}-\frac{4}{n^4}}\right)=\lim_{n\to\infty}\frac{1}{n}\cdot\lim_{n\to\infty}\left(\frac{2+\frac{3}{n^3}}{1+\frac{2}{n^2}-\frac{4}{n^4}}\right)=0\cdot\lim_{n\to\infty}\left(\frac{2+\frac{3}{n^3}}{1+\frac{2}{n^2}-\frac{4}{n^4}}\right)=0$$

зад.6. Пресметнете: $\lim_{n\to\infty}\left(\frac{n^2}{2n-1}-\frac{n^2+1}{2n+1}\right)=?$

$$\lim_{n\to\infty}\left(\frac{n^2}{2n-1}-\frac{n^2+1}{2n+1}\right)=\lim_{n\to\infty}\left(\frac{n^2(2n+1)}{(2n-1)(2n+1)}-\frac{(n^2+1)(2n-1)}{(2n-1)(2n+1)}\right)=$$

$$=\lim_{n\to\infty}\left(\frac{n^2(2n+1)-(n^2+1)(2n-1)}{(2n-1)(2n+1)}\right)=\lim_{n\to\infty}\left(\frac{2n^3+n^2-2n^3+n^2-2n+1}{(2n-1)(2n+1)}\right)=$$

$$=\lim_{n\to\infty}\left(\frac{2n^2-2n+1}{4n^2-1}\right)=\lim_{n\to\infty}\frac{n^2\left(2-\frac{2}{n}+\frac{1}{n^2}\right)}{n^2\left(4-\frac{1}{n^2}\right)}=\lim_{n\to\infty}\frac{\left(2-\frac{2}{n}+\frac{1}{n^2}\right)}{\left(4-\frac{1}{n^2}\right)}=$$

$$=\frac{\lim_{n\to\infty}\left(2-\frac{2}{n}+\frac{1}{n^2}\right)}{\lim_{n\to\infty}\left(4-\frac{1}{n^2}\right)}=\frac{\lim_{n\to\infty}2-\lim_{n\to\infty}\frac{2}{n}+\lim_{n\to\infty}\frac{1}{n^2}}{\lim_{n\to\infty}4-\lim_{n\to\infty}\frac{1}{n^2}}=\frac{2-0+0}{4-0}=\frac{2}{4}=\frac{1}{2}$$

зад.7. Пресметнете: $\lim_{n\to\infty}\frac{(n+1)^3-(n-1)^3}{2n^2+2n+1}=?$

$$\lim_{n\to\infty}\frac{(n+1)^3-(n-1)^3}{2n^2+2n+1}=\lim_{n\to\infty}\frac{n^3+3n^2+3n+1-\left(n^3-3n^2+3n-1\right)}{2n^2+2n+1}=$$

$$=\lim_{n\to\infty}\frac{n^3+3n^2+3n+1-n^3+3n^2-3n+1}{2n^2+2n+1}=\lim_{n\to\infty}\frac{6n^2+2}{2n^2+2n+1}=$$

$$=\lim_{n\to\infty}\frac{n^2\left(6+\frac{2}{n^2}\right)}{n^2\left(2+\frac{2}{n}+\frac{1}{n^2}\right)}=\lim_{n\to\infty}\frac{\left(6+\frac{2}{n^2}\right)}{\left(2+\frac{2}{n}+\frac{1}{n^2}\right)}=\frac{\lim_{n\to\infty}\left(6+\frac{2}{n^2}\right)}{\lim_{n\to\infty}\left(2+\frac{2}{n}+\frac{1}{n^2}\right)}=$$

$$=\frac{\lim_{n\to\infty}6+\lim_{n\to\infty}\frac{2}{n^2}}{\lim_{n\to\infty}2+\lim_{n\to\infty}\frac{2}{n}+\lim_{n\to\infty}\frac{1}{n^2}}=\frac{6+0}{2+0+0}=\frac{6}{2}=3$$

зад.8. Пресметнете границата, като предварително съкратите дробта:

$$\lim_{a_n \to 2} \frac{a_n^2 - 4}{a_n - 2} = ?$$

$$\lim_{a_n \to 2} \frac{a_n^2 - 4}{a_n - 2} = \lim_{a_n \to 2} \frac{(a_n - 2)(a_n + 2)}{a_n - 2} = \lim_{a_n \to 2} (a_n + 2) =$$

$$= \lim_{a_n \to 2} a_n + 2 = 2 + 2 = 4$$

зад.9. Пресметнете границата, като предварително съкратите дробта:

$$\lim_{a_n\to 3}\frac{a_n^2-5a_n+6}{a_n-3}=?$$

$$\lim_{a_n\to 3}\frac{a_n^2-5a_n+6}{a_n-3}=\lim_{a_n\to 3}\frac{a_n^2-3a_n-2a_n+6}{a_n-3}=$$

$$=\lim_{a_n\to 3}\frac{\left(a_n^2-3a_n\right)-\left(2a_n-6\right)}{a_n-3}=\lim_{a_n\to 3}\frac{a_n\left(a_n-3\right)-2\left(a_n-3\right)}{a_n-3}$$

$$=\lim_{a_n\to 3}\frac{\left(a_n-2\right)\left(a_n-3\right)}{a_n-3}=\lim_{a_n\to 3}\left(a_n-2\right)=\lim_{a_n\to 3}a_n-2=3-2=1$$

зад.10. Пресметнете границата, като предварително съкратите дробта:

$$\lim_{a_n \to -3} \frac{a_n^2 - 9}{a_n^2 + 2a_n - 3} = ?$$

$$\lim_{a_n \to -3} \frac{a_n^2 - 9}{a_n^2 + 2a_n - 3} = \lim_{a_n \to -3} \frac{(a_n - 3)(a_n + 3)}{a_n^2 + 3a_n - a_n - 3} = \lim_{a_n \to -3} \frac{(a_n - 3)(a_n + 3)}{(a_n^2 + 3a_n) - (a_n + 3)} =$$

$$= \lim_{a_n \to -3} \frac{(a_n - 3)(a_n + 3)}{a_n(a_n + 3) - (a_n + 3)} = \lim_{a_n \to -3} \frac{(a_n - 3)(a_n + 3)}{(a_n + 3)(a_n - 1)} = \lim_{a_n \to -3} \frac{(a_n - 3)}{(a_n - 1)} =$$

$$= \frac{\lim_{a_n \to -3}(a_n - 3)}{\lim_{a_n \to -3}(a_n - 1)} = \frac{\lim_{a_n \to -3} a_n - 3}{\lim_{a_n \to -3} a_n - 1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$$

зад.11. Пресметнете границата, като предварително съкратите дробта:

$$\lim_{a_n \to a} \frac{a_n^2 - a^2}{a_n - a} = ?$$

$$\lim_{a_n \to a} \frac{a_n^2 - a^2}{a_n - a} = \lim_{a_n \to a} \frac{(a_n - a)(a_n + a)}{a_n - a} = \lim_{a_n \to a} (a_n + a) =$$

$$= \lim_{a_n \to a} a_n + a = a + a = 2a$$